Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.2-Cosine/86-4.2.1.2-g-sin-^p-a+b-cos-^m

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [88]. This is test number [86].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (88)	0.00 (0)
Mathematica	100.00 (88)	0.00 (0)
Maple	100.00 (88)	0.00 (0)
Fricas	64.77 (57)	35.23 (31)
Mupad	38.64 (34)	61.36 (54)
Giac	36.36 (32)	63.64 (56)
Maxima	30.68 (27)	69.32 (61)
Sympy	26.14 (23)	73.86 (65)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

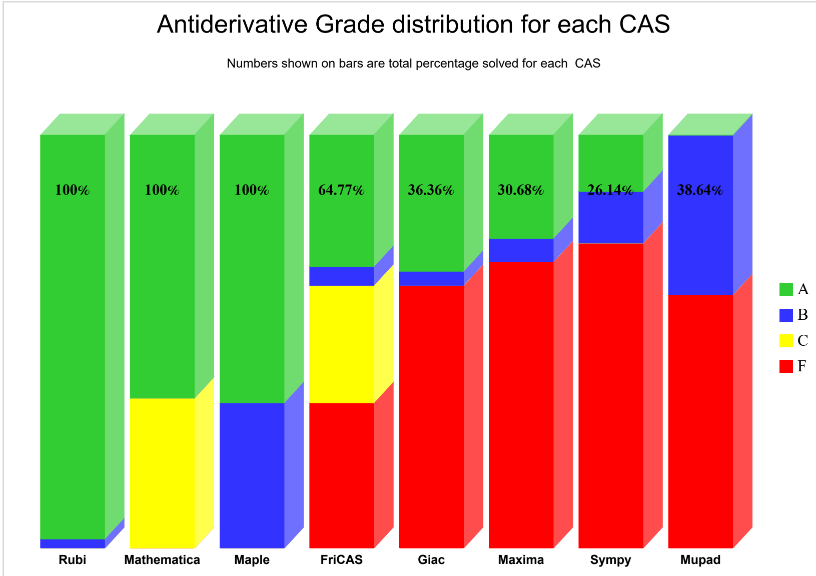
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

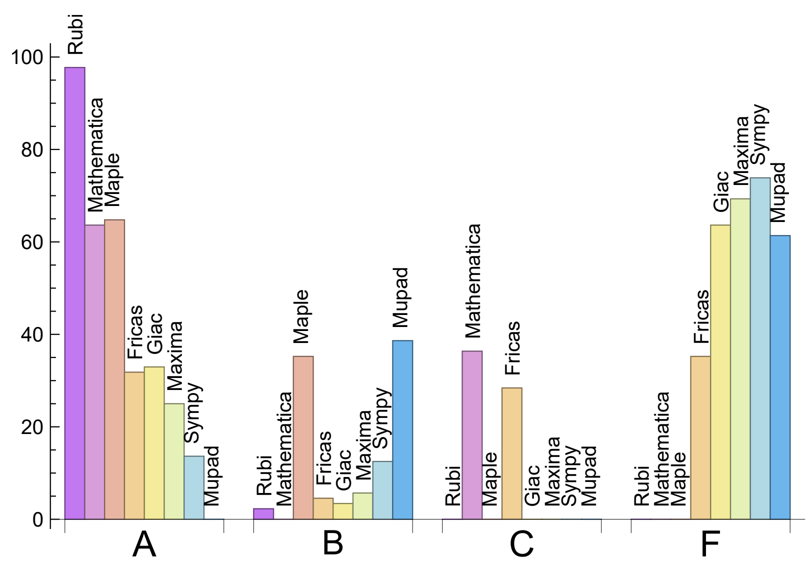
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.73	2.27	0.00	0.00
Maple	64.77	35.23	0.00	0.00
Mathematica	63.64	0.00	36.36	0.00
Giac	32.95	3.41	0.00	63.64
Fricas	31.82	4.55	28.41	35.23
Maxima	25.00	5.68	0.00	69.32
Sympy	13.64	12.50	0.00	73.86
Mupad	N/A	38.64	0.00	61.36

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	31	16.13 %	83.87 %	0.00 %
Giac	56	82.14 %	17.86 %	0.00 %
Maxima	61	73.77 %	18.03 %	8.20 %
Sympy	65	47.69 %	36.92 %	15.38 %
Mupad	54	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

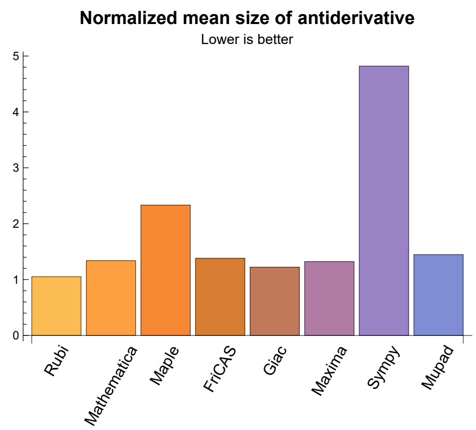
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

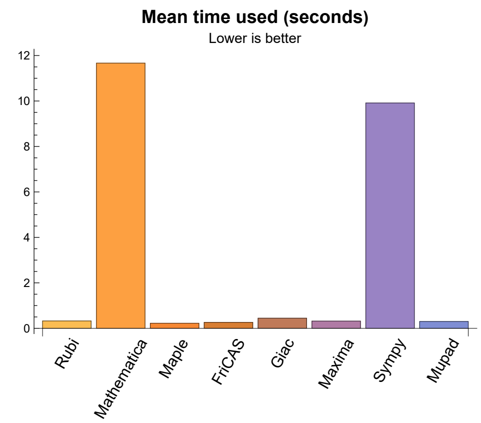
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.32	226.90	1.05	142.50	1.00
Mathematica	11.67	408.16	1.34	101.50	1.04
Maple	0.22	819.76	2.33	227.50	1.67
Maxima	0.32	27.85	1.32	14.00	1.00
Fricas	0.26	104.02	1.38	104.00	1.14
Sympy	9.91	149.30	4.82	15.00	2.00
Giac	0.44	40.88	1.22	14.00	1.01
Mupad	0.30	80.32	1.45	13.00	0.93

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
12
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

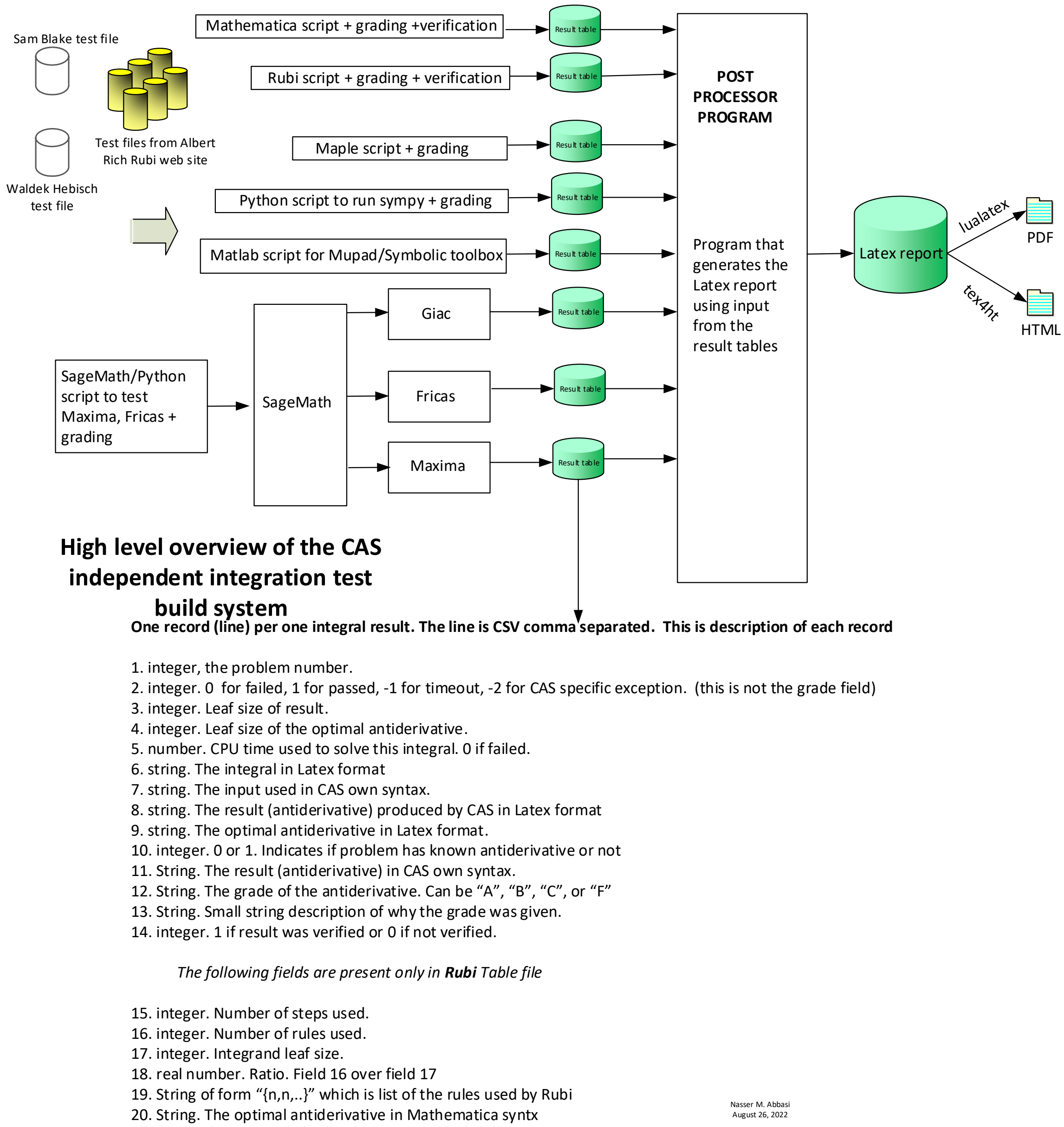
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

B grade: { 10, 11 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57 }

B grade: { }

C grade: { 15, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 63, 64, 65, 66 }

B grade: { 3, 11, 44, 46, 58, 59, 60, 61, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

C grade: { }

F grade: { }

2.1.4 Maxima

A grade: { 2, 4, 5, 6, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 27, 29, 31 }

B grade: { 1, 3, 7, 9, 11 }

C grade: { }

F grade: { 24, 26, 28, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30 }

B grade: { 10, 11, 31, 32 }

C grade: { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57 }

F grade: { 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

2.1.6 Sympy

A grade: { 4, 5, 10, 12, 13, 14, 15, 18, 19, 20, 21, 27 }

B grade: { 1, 2, 3, 11, 16, 17, 22, 23, 25, 26, 28 }

C grade: { }

F grade: { 6, 7, 8, 9, 24, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31 }

B grade: { 11, 24, 32 }

C grade: { }

F grade: { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37 }

C grade: { }

F grade: { 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	48	94	24	294	45	34
N.S.	1	1.00	0.81	1.55	3.03	0.77	9.48	1.45	1.10
time (sec)	N/A	0.028	0.051	0.079	0.493	0.367	0.515	0.395	0.382

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	13	16	14	14	51	14	11
N.S.	1	1.00	0.68	0.84	0.74	0.74	2.68	0.74	0.58
time (sec)	N/A	0.026	0.014	0.053	0.283	0.365	0.300	0.414	0.249

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	17	30	42	10	46	25	10
N.S.	1	1.00	1.31	2.31	3.23	0.77	3.54	1.92	0.77
time (sec)	N/A	0.026	0.013	0.056	0.510	0.370	0.187	0.461	0.292

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	13	12	12	8	10	10
N.S.	1	1.00	1.20	1.30	1.20	1.20	0.80	1.00	1.00
time (sec)	N/A	0.015	0.009	0.043	0.300	0.363	0.053	0.414	0.057

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	9	12	11	5	8	8
N.S.	1	1.00	0.91	0.82	1.09	1.00	0.45	0.73	0.73
time (sec)	N/A	0.010	0.007	0.029	0.269	0.348	0.091	0.419	0.293

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	42	28	31	37	0	34	20
N.S.	1	1.00	1.83	1.22	1.35	1.61	0.00	1.48	0.87
time (sec)	N/A	0.033	0.038	0.074	0.286	0.366	0.000	0.480	0.285

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	29	41	26	0	37	35
N.S.	1	1.00	1.25	1.21	1.71	1.08	0.00	1.54	1.46
time (sec)	N/A	0.030	0.056	0.066	0.262	0.346	0.000	0.407	0.320

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	60	44	58	83	0	52	45
N.S.	1	1.00	1.22	0.90	1.18	1.69	0.00	1.06	0.92
time (sec)	N/A	0.052	0.130	0.105	0.266	0.375	0.000	0.396	0.289

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	38	45	70	53	0	59	45
N.S.	1	1.00	1.03	1.22	1.89	1.43	0.00	1.59	1.22
time (sec)	N/A	0.032	0.066	0.088	0.297	0.354	0.000	0.556	0.372

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	11	5	10	9	11	10	9	7
N.S.	1	2.20	1.00	2.00	1.80	2.20	2.00	1.80	1.40
time (sec)	N/A	0.015	0.005	0.037	0.314	0.375	0.039	0.422	0.264

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	13	3	12	9	11	8	11	9
N.S.	1	4.33	1.00	4.00	3.00	3.67	2.67	3.67	3.00
time (sec)	N/A	0.016	0.006	0.038	0.271	0.368	0.041	0.416	0.077

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	12	7	6	6	5	6	6
N.S.	1	1.00	2.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.014	0.011	0.036	0.262	0.353	0.153	0.404	0.251

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	11	6	6	5	6	6
N.S.	1	1.00	1.20	1.10	0.60	0.60	0.50	0.60	0.60
time (sec)	N/A	0.013	0.012	0.046	0.290	0.380	0.161	0.446	0.248

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	23	18	7	10	10
N.S.	1	1.00	1.29	1.07	1.64	1.29	0.50	0.71	0.71
time (sec)	N/A	0.021	0.008	0.051	0.480	0.347	0.209	0.410	0.298

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	17	23	16	8	12	10
N.S.	1	1.00	1.62	1.06	1.44	1.00	0.50	0.75	0.62
time (sec)	N/A	0.021	0.014	0.076	0.533	0.373	0.373	0.422	0.305

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	13	11	10	12	58	10	10
N.S.	1	1.00	1.30	1.10	1.00	1.20	5.80	1.00	1.00
time (sec)	N/A	0.023	0.022	0.059	0.271	0.398	0.211	0.405	0.247

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	13	11	10	12	58	12	10
N.S.	1	1.00	1.08	0.92	0.83	1.00	4.83	1.00	0.83
time (sec)	N/A	0.024	0.020	0.076	0.284	0.373	0.202	0.386	0.044

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	8	14	14	8	8
N.S.	1	1.00	1.20	0.90	0.80	1.40	1.40	0.80	0.80
time (sec)	N/A	0.013	0.010	0.040	0.324	0.363	0.256	0.489	0.035

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	8	14	15	8	8
N.S.	1	1.00	1.00	0.92	0.67	1.17	1.25	0.67	0.67
time (sec)	N/A	0.013	0.013	0.054	0.296	0.354	0.239	0.454	0.036

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	12	20	7	8	8
N.S.	1	1.00	0.86	0.64	0.86	1.43	0.50	0.57	0.57
time (sec)	N/A	0.020	0.031	0.059	0.263	0.346	0.341	0.421	0.259

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	9	12	22	10	8	8
N.S.	1	1.00	0.75	0.56	0.75	1.38	0.62	0.50	0.50
time (sec)	N/A	0.021	0.038	0.086	0.269	0.343	0.542	0.521	0.353

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	14	21	126	14	14
N.S.	1	1.00	1.29	1.07	1.00	1.50	9.00	1.00	1.00
time (sec)	N/A	0.025	0.011	0.069	0.265	0.359	0.269	0.438	0.043

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	29	17	16	22	126	18	16
N.S.	1	1.00	1.45	0.85	0.80	1.10	6.30	0.90	0.80
time (sec)	N/A	0.024	0.013	0.091	0.277	0.364	0.246	0.427	0.044

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	96	152	0	243	0	194	1677
N.S.	1	1.00	0.92	1.46	0.00	2.34	0.00	1.87	16.12
time (sec)	N/A	0.175	0.224	0.169	0.000	0.402	0.000	0.514	1.112

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	38	41	1421	39	38
N.S.	1	1.00	1.00	0.98	0.95	1.02	35.52	0.98	0.95
time (sec)	N/A	0.041	0.060	0.084	0.264	0.390	167.896	0.430	0.092

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	78	0	154	991	90	74
N.S.	1	1.00	0.92	1.32	0.00	2.61	16.80	1.53	1.25
time (sec)	N/A	0.075	0.085	0.107	0.000	0.405	53.722	0.454	0.488

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	15	17	13	12
N.S.	1	1.00	1.00	1.08	1.00	1.25	1.42	1.08	1.00
time (sec)	N/A	0.016	0.018	0.043	0.271	0.365	0.159	0.504	0.043

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	41	36	0	137	144	61	38
N.S.	1	1.00	0.98	0.86	0.00	3.26	3.43	1.45	0.90
time (sec)	N/A	0.017	0.027	0.063	0.000	0.379	1.793	0.436	0.483

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	54	47	52	0	54	52
N.S.	1	1.00	0.94	1.02	0.89	0.98	0.00	1.02	0.98
time (sec)	N/A	0.050	0.047	0.097	0.295	0.419	0.000	0.422	0.212

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	66	78	0	230	0	91	86
N.S.	1	1.00	0.99	1.16	0.00	3.43	0.00	1.36	1.28
time (sec)	N/A	0.068	0.353	0.131	0.000	0.395	0.000	0.554	0.472

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	99	96	115	181	0	136	112
N.S.	1	1.00	1.08	1.04	1.25	1.97	0.00	1.48	1.22
time (sec)	N/A	0.107	0.464	0.164	0.373	0.419	0.000	0.472	0.508

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	112	127	0	459	0	206	184
N.S.	1	1.00	1.02	1.15	0.00	4.17	0.00	1.87	1.67
time (sec)	N/A	0.179	0.702	0.213	0.000	0.385	0.000	0.444	0.558

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	108	127	0	130	0	0	-1
N.S.	1	1.00	0.84	0.98	0.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.906	0.145	0.000	0.121	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	80	171	0	110	0	0	-1
N.S.	1	1.00	0.80	1.71	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.555	0.132	0.000	0.125	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	80	116	0	104	0	0	-1
N.S.	1	1.00	0.80	1.16	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.496	0.128	0.000	0.105	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	60	117	0	83	0	0	60
N.S.	1	1.00	0.88	1.72	0.00	1.22	0.00	0.00	0.88
time (sec)	N/A	0.034	0.110	0.128	0.000	0.102	0.000	0.000	0.494

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	54	92	0	70	0	0	50
N.S.	1	1.00	0.82	1.39	0.00	1.06	0.00	0.00	0.76
time (sec)	N/A	0.034	0.218	0.195	0.000	0.110	0.000	0.000	0.724

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	58	153	0	107	0	0	-1
N.S.	1	1.00	0.60	1.59	0.00	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.128	0.122	0.000	0.098	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	59	124	0	127	0	0	-1
N.S.	1	1.00	0.58	1.22	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.168	0.126	0.000	0.100	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	74	187	0	171	0	0	-1
N.S.	1	1.00	0.56	1.43	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.277	0.135	0.000	0.103	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	157	252	0	188	0	0	-1
N.S.	1	1.00	0.81	1.31	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.131	1.733	0.145	0.000	0.134	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	116	332	0	157	0	0	-1
N.S.	1	1.00	0.75	2.16	0.00	1.02	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.840	0.152	0.000	0.149	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	117	229	0	151	0	0	-1
N.S.	1	1.00	0.76	1.49	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.873	0.132	0.000	0.120	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	83	294	0	120	0	0	-1
N.S.	1	1.00	0.73	2.58	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.298	0.145	0.000	0.106	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	79	170	0	105	0	0	-1
N.S.	1	1.00	0.69	1.49	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.418	0.115	0.000	0.102	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	75	283	0	134	0	0	-1
N.S.	1	1.00	0.64	2.40	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.257	0.119	0.000	0.105	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	76	202	0	168	0	0	-1
N.S.	1	1.00	0.61	1.63	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.291	0.130	0.000	0.102	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	109	351	0	225	0	0	-1
N.S.	1	1.00	0.66	2.13	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.505	0.137	0.000	0.118	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	205	276	0	238	0	0	-1
N.S.	1	1.00	0.85	1.14	0.00	0.98	0.00	0.00	-0.00
time (sec)	N/A	0.214	2.581	0.176	0.000	0.160	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	149	356	0	194	0	0	-1
N.S.	1	1.00	0.74	1.76	0.00	0.96	0.00	0.00	-0.00
time (sec)	N/A	0.190	1.461	0.178	0.000	0.181	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	147	291	0	190	0	0	-1
N.S.	1	1.00	0.73	1.44	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.191	1.299	0.171	0.000	0.135	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	105	315	0	148	0	0	-1
N.S.	1	1.00	0.65	1.96	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.594	0.161	0.000	0.118	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	98	210	0	126	0	0	-1
N.S.	1	1.00	0.62	1.34	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.774	0.131	0.000	0.114	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	101	313	0	162	0	0	-1
N.S.	1	1.00	0.61	1.90	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.365	0.142	0.000	0.139	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	102	226	0	196	0	0	-1
N.S.	1	1.00	0.60	1.34	0.00	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.922	0.139	0.000	0.114	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	130	375	0	251	0	0	-1
N.S.	1	1.00	0.68	1.95	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.668	0.154	0.000	0.112	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	144	265	0	287	0	0	-1
N.S.	1	1.00	0.75	1.37	0.00	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.670	0.158	0.000	0.108	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	2035	2048	0	0	0	0	-1
N.S.	1	1.00	3.74	3.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.255	47.419	0.324	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	834	1473	0	0	0	0	-1
N.S.	1	1.00	1.81	3.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.835	35.036	0.271	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	1955	1534	0	0	0	0	-1
N.S.	1	1.00	4.12	3.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.863	45.912	0.243	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	757	968	0	0	0	0	-1
N.S.	1	1.00	1.90	2.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.564	34.451	0.207	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	434	1068	0	0	0	0	-1
N.S.	1	1.00	1.06	2.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.579	15.865	0.196	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	361	545	0	0	0	0	-1
N.S.	1	1.00	1.20	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	11.804	0.163	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	261	600	0	0	0	0	-1
N.S.	1	1.00	0.85	1.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.368	12.029	0.174	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	791	843	0	0	0	0	-1
N.S.	1	1.00	1.86	1.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.628	34.766	0.185	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	1192	820	0	0	0	0	-1
N.S.	1	1.00	2.67	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.667	31.775	0.255	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	881	1080	0	0	0	0	-1
N.S.	1	1.00	1.76	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.869	26.574	0.238	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	2029	2510	0	0	0	0	-1
N.S.	1	1.00	3.64	4.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.030	45.173	0.465	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	835	1995	0	0	0	0	-1
N.S.	1	1.00	1.77	4.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.742	34.728	0.415	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	1956	1964	0	0	0	0	-1
N.S.	1	1.00	4.02	4.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.740	44.561	0.373	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	366	1722	0	0	0	0	-1
N.S.	1	1.00	0.91	4.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.529	80.521	0.247	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	614	1460	0	0	0	0	-1
N.S.	1	1.00	1.47	3.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.577	19.534	0.361	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	786	1398	0	0	0	0	-1
N.S.	1	1.00	1.79	3.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.555	34.072	0.308	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	445	445	1182	1369	0	0	0	0	-1
N.S.	1	1.00	2.66	3.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.578	30.695	0.331	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	507	507	865	2081	0	0	0	0	-1
N.S.	1	1.00	1.71	4.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.811	26.482	0.300	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	1257	1599	0	0	0	0	-1
N.S.	1	1.00	2.37	3.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.902	33.710	0.451	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	590	590	950	1837	0	0	0	0	-1
N.S.	1	1.00	1.61	3.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.079	26.603	0.491	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	590	590	930	3821	0	0	0	0	-1
N.S.	1	1.00	1.58	6.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.928	35.161	0.758	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	604	604	2024	3726	0	0	0	0	-1
N.S.	1	1.00	3.35	6.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.020	44.742	0.716	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	837	3198	0	0	0	0	-1
N.S.	1	1.00	1.68	6.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.697	34.492	0.607	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	946	3146	0	0	0	0	-1
N.S.	1	1.00	1.85	6.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.709	38.826	0.638	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	831	3101	0	0	0	0	-1
N.S.	1	1.00	1.60	5.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.724	32.842	0.600	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1211	3065	0	0	0	0	-1
N.S.	1	1.00	2.27	5.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.761	29.161	0.611	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	748	2890	0	0	0	0	-1
N.S.	1	1.00	1.41	5.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.759	21.769	0.551	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	535	535	1226	2838	0	0	0	0	-1
N.S.	1	1.00	2.29	5.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.803	29.748	0.594	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	611	611	922	3431	0	0	0	0	-1
N.S.	1	1.00	1.51	5.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.068	16.539	0.713	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	629	629	1308	3374	0	0	0	0	-1
N.S.	1	1.00	2.08	5.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.147	34.992	0.810	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	700	700	1014	3709	0	0	0	0	-1
N.S.	1	1.00	1.45	5.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.353	16.652	0.873	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [41] had the largest ratio of [25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	13	0.231
2	A	2	1	1.00	13	0.077
3	A	2	2	1.00	13	0.154
4	A	2	2	1.00	11	0.182
5	A	1	1	1.00	8	0.125
6	A	4	3	1.00	11	0.273
7	A	3	3	1.00	13	0.231
8	A	4	3	1.00	13	0.231
9	A	3	2	1.00	13	0.154
10	B	2	2	2.20	13	0.154
11	B	2	2	4.33	15	0.133
12	A	2	2	1.00	9	0.222
13	A	2	2	1.00	11	0.182
14	A	2	2	1.00	11	0.182
15	A	2	2	1.00	13	0.154
16	A	3	2	1.00	11	0.182
17	A	3	2	1.00	13	0.154
18	A	2	2	1.00	9	0.222
19	A	2	2	1.00	11	0.182
20	A	1	1	1.00	11	0.091
21	A	1	1	1.00	13	0.077
22	A	3	2	1.00	11	0.182
23	A	3	2	1.00	13	0.154
24	A	5	5	1.00	13	0.385
25	A	3	2	1.00	13	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	4	1.00	13	0.308
27	A	2	2	1.00	11	0.182
28	A	2	2	1.00	8	0.250
29	A	6	4	1.00	11	0.364
30	A	4	4	1.00	13	0.308
31	A	4	3	1.00	13	0.231
32	A	5	5	1.00	13	0.385
33	A	5	4	1.00	23	0.174
34	A	4	4	1.00	23	0.174
35	A	4	4	1.00	23	0.174
36	A	3	3	1.00	23	0.130
37	A	3	3	1.00	23	0.130
38	A	4	4	1.00	23	0.174
39	A	4	4	1.00	23	0.174
40	A	5	4	1.00	23	0.174
41	A	6	5	1.00	25	0.200
42	A	5	5	1.00	25	0.200
43	A	5	5	1.00	25	0.200
44	A	4	4	1.00	25	0.160
45	A	4	4	1.00	25	0.160
46	A	4	4	1.00	25	0.160
47	A	4	4	1.00	25	0.160
48	A	5	5	1.00	25	0.200
49	A	7	6	1.00	25	0.240
50	A	6	6	1.00	25	0.240
51	A	6	6	1.00	25	0.240
52	A	5	5	1.00	25	0.200
53	A	5	5	1.00	25	0.200
54	A	5	5	1.00	25	0.200
55	A	5	5	1.00	25	0.200
56	A	5	5	1.00	25	0.200
57	A	5	5	1.00	25	0.200
58	A	15	12	1.00	25	0.480
59	A	14	12	1.00	25	0.480
60	A	14	12	1.00	25	0.480

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	13	11	1.00	25	0.440
62	A	13	11	1.00	25	0.440
63	A	9	7	1.00	25	0.280
64	A	9	7	1.00	25	0.280
65	A	13	11	1.00	25	0.440
66	A	13	11	1.00	25	0.440
67	A	14	12	1.00	25	0.480
68	A	15	12	1.00	25	0.480
69	A	14	12	1.00	25	0.480
70	A	14	12	1.00	25	0.480
71	A	13	11	1.00	25	0.440
72	A	13	11	1.00	25	0.440
73	A	13	11	1.00	25	0.440
74	A	13	11	1.00	25	0.440
75	A	14	12	1.00	25	0.480
76	A	14	12	1.00	25	0.480
77	A	15	12	1.00	25	0.480
78	A	15	13	1.00	25	0.520
79	A	15	13	1.00	25	0.520
80	A	14	12	1.00	25	0.480
81	A	14	12	1.00	25	0.480
82	A	14	12	1.00	25	0.480
83	A	14	12	1.00	25	0.480
84	A	14	12	1.00	25	0.480
85	A	14	12	1.00	25	0.480
86	A	15	13	1.00	25	0.520
87	A	15	13	1.00	25	0.520
88	A	16	13	1.00	25	0.520

Chapter 3

Listing of integrals

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3.65 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$ 300
3.66 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$ 306
3.67 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$ 313
3.68 $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$ 320
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3.88 $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$ 469

3.1 $\int \frac{\sin^4(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=31

$$\frac{x}{2a}-\frac{\cos(x)\sin(x)}{2a}-\frac{\sin^3(x)}{3a}$$

```
[Out] 1/2*x/a-1/2*cos(x)*sin(x)/a-1/3*sin(x)^3/a
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2761, 2715, 8}

$$\frac{x}{2a}-\frac{\sin^3(x)}{3a}-\frac{\sin(x)\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + a*Cos[x]),x]
```

```
[Out] x/(2*a) - (Cos[x]*Sin[x])/(2*a) - Sin[x]^3/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sin^4(x)}{a + a\cos(x)}\,dx = -\frac{\sin^3(x)}{3a} + \frac{\int \sin^2(x)\,dx}{a}$$
$$= -\frac{\cos(x)\sin(x)}{2a} - \frac{\sin^3(x)}{3a} + \frac{\int 1\,dx}{2a}$$
$$= \frac{x}{2a} - \frac{\cos(x)\sin(x)}{2a} - \frac{\sin^3(x)}{3a}$$

Mathematica [A]
time = 0.05, size = 25, normalized size = 0.81

$$\frac{6x - 3\sin(x) - 3\sin(2x) + \sin(3x)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + a*Cos[x]),x]
[Out] (6*x - 3*Sin[x] - 3*Sin[2*x] + Sin[3*x])/(12*a)
```

Maple [A]
time = 0.08, size = 48, normalized size = 1.55

method	result	size
risch	$\frac{x}{2a} - \frac{\sin(x)}{4a} + \frac{\sin(3x)}{12a} - \frac{\sin(2x)}{4a}$	33
default	$\frac{\frac{16\left(\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{16} - \frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{6} - \frac{\tan\left(\frac{x}{2}\right)}{16}\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3} + \arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$	48
norman	$\frac{\frac{\tan^7\left(\frac{x}{2}\right)}{a} - \frac{\tan\left(\frac{x}{2}\right)}{a} - \frac{11\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a} - \frac{5\left(\tan^5\left(\frac{x}{2}\right)\right)}{3a} + \frac{x}{2a} + \frac{2x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a} + \frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a} + \frac{2x\left(\tan^6\left(\frac{x}{2}\right)\right)}{a} + \frac{x\left(\tan^8\left(\frac{x}{2}\right)\right)}{2a}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^4}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+a*cos(x)),x,method=_RETURNVERBOSE)
[Out] 16/a*((1/16*tan(1/2*x)^5-1/6*tan(1/2*x)^3-1/16*tan(1/2*x))/(tan(1/2*x)^2+1)
^3+1/16*arctan(tan(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(25) = 50.
time = 0.49, size = 94, normalized size = 3.03

$$-\frac{\frac{3\,\sin(x)}{\cos(x)+1} + \frac{8\,\sin(x)^3}{(\cos(x)+1)^3} - \frac{3\,\sin(x)^5}{(\cos(x)+1)^5}}{3\left(a + \frac{3\,a\sin(x)^2}{(\cos(x)+1)^2} + \frac{3\,a\sin(x)^4}{(\cos(x)+1)^4} + \frac{a\sin(x)^6}{(\cos(x)+1)^6}\right)} + \frac{\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sin(x)/(cos(x) + 1) + 8*sin(x)^3/(cos(x) + 1)^3 - 3*sin(x)^5/(cos(x
) + 1)^5)/(a + 3*a*sin(x)^2/(cos(x) + 1)^2 + 3*a*sin(x)^4/(cos(x) + 1)^4 +
a*sin(x)^6/(cos(x) + 1)^6) + arctan(sin(x)/(cos(x) + 1))/a
```

Fricas [A]
time = 0.37, size = 24, normalized size = 0.77

$$\frac{\left(2\,\cos\left(x\right)^{2}-3\,\cos\left(x\right)-2\right)\sin\left(x\right)+3\,x}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/6*((2*cos(x)^2 - 3*cos(x) - 2)*sin(x) + 3*x)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(22) = 44$.
time = 0.52, size = 294, normalized size = 9.48

$$\frac{3x\tan^{6}\left(\frac{x}{2}\right)}{6a\tan^{6}\left(\frac{x}{2}\right)+18a\tan^{4}\left(\frac{x}{2}\right)+18a\tan^{2}\left(\frac{x}{2}\right)+6a}+\frac{9x\tan^{4}\left(\frac{x}{2}\right)}{6a\tan^{6}\left(\frac{x}{2}\right)+18a\tan^{4}\left(\frac{x}{2}\right)+18a\tan^{2}\left(\frac{x}{2}\right)+6a}+\frac{9x\tan^{2}\left(\frac{x}{2}\right)}{6a\tan^{6}\left(\frac{x}{2}\right)+18a\tan^{4}\left(\frac{x}{2}\right)+18a\tan^{2}\left(\frac{x}{2}\right)+6a}+\frac{3x}{6a\tan^{6}\left(\frac{x}{2}\right)+18a\tan^{4}\left(\frac{x}{2}\right)+18a\tan^{2}\left(\frac{x}{2}\right)+6a}+\frac{6\tan^{5}\left(\frac{x}{2}\right)}{6a\tan^{6}\left(\frac{x}{2}\right)+18a\tan^{4}\left(\frac{x}{2}\right)+18a\tan^{2}\left(\frac{x}{2}\right)+6a}-\frac{16\tan^{3}\left(\frac{x}{2}\right)}{6a\tan^{6}\left(\frac{x}{2}\right)+18a\tan^{4}\left(\frac{x}{2}\right)+18a\tan^{2}\left(\frac{x}{2}\right)+6a}-\frac{6\tan\left(\frac{x}{2}\right)}{6a\tan^{6}\left(\frac{x}{2}\right)+18a\tan^{4}\left(\frac{x}{2}\right)+18a\tan^{2}\left(\frac{x}{2}\right)+6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+a*cos(x)),x)
```

```
[Out] 3*x*tan(x/2)**6/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**2 + 6*
a) + 9*x*tan(x/2)**4/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**2
 + 6*a) + 9*x*tan(x/2)**2/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/
2)**2 + 6*a) + 3*x/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**2 +
 6*a) + 6*tan(x/2)**5/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/2)**
2 + 6*a) - 16*tan(x/2)**3/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/
2)**2 + 6*a) - 6*tan(x/2)/(6*a*tan(x/2)**6 + 18*a*tan(x/2)**4 + 18*a*tan(x/
2)**2 + 6*a)
```

Giac [A]
time = 0.39, size = 45, normalized size = 1.45

$$\frac{x}{2\,a}+\frac{3\,\tan\left(\frac{1}{2}\,x\right)^{5}-8\,\tan\left(\frac{1}{2}\,x\right)^{3}-3\,\tan\left(\frac{1}{2}\,x\right)}{3\,{\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)}^{3}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] 1/2*x/a + 1/3*(3*tan(1/2*x)^5 - 8*tan(1/2*x)^3 - 3*tan(1/2*x))/((tan(1/2*x)
^2 + 1)^3*a)
```

Mupad [B]
time = 0.38, size = 34, normalized size = 1.10

$$\frac{x}{2\,a}-\frac{\sin\left(x\right)}{3\,a}+\frac{{\cos\left(x\right)}^2\,\sin\left(x\right)}{3\,a}-\frac{\cos\left(x\right)\,\sin\left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + a*cos(x)),x)
[Out] x/(2*a) - sin(x)/(3*a) + (cos(x)^2*sin(x))/(3*a) - (cos(x)*sin(x))/(2*a)
```

3.2 $\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{\cos(x)}{a}+\frac{\cos^2(x)}{2a}$$

```
[Out] -cos(x)/a+1/2*cos(x)^2/a
```

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{2746\}$

$$\frac{\cos^2(x)}{2a}-\frac{\cos(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + a*Cos[x]),x]
```

```
[Out] -(Cos[x]/a) + Cos[x]^2/(2*a)
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(x)}{a+a\cos(x)}\,dx &= -\frac{\text{Subst}\left(\int (a-x)\,dx,x,a\cos(x)\right)}{a^3}\\ &= -\frac{\cos(x)}{a}+\frac{\cos^2(x)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 0.68

$$\frac{2\sin^4\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + a*Cos[x]),x]
[Out] (2*Sin[x/2]^4)/a
```

Maple [A]
time = 0.05, size = 16, normalized size = 0.84

method	result	size
derivativedivides	$\frac{\frac{\left(\cos^{2}\left(x\right)\right)}{2}-\cos\left(x\right)}{a}$	16
default	$\frac{\frac{\left(\cos^{2}\left(x\right)\right)}{2}-\cos\left(x\right)}{a}$	16
risch	$-\frac{\cos\left(x\right)}{a}+\frac{\cos\left(2x\right)}{4a}$	18
norman	$\frac{-\frac{2}{a}-\frac{4\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{a}-\frac{6\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a}}{\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)^{3}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+a*cos(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(1/2*cos(x)^2-cos(x))
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.74

$$\frac{\cos\left(x\right)^{2}-2\,\cos\left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*cos(x)),x, algorithm="maxima")
[Out] 1/2*(cos(x)^2 - 2*cos(x))/a
```

Fricas [A]
time = 0.36, size = 14, normalized size = 0.74

$$\frac{\cos\left(x\right)^{2}-2\,\cos\left(x\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*cos(x)),x, algorithm="fricas")
[Out] 1/2*(cos(x)^2 - 2*cos(x))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(12) = 24$.
time = 0.30, size = 51, normalized size = 2.68

$$-\frac{4\tan^{2}\left(\frac{x}{2}\right)}{a\tan^{4}\left(\frac{x}{2}\right)+2a\tan^{2}\left(\frac{x}{2}\right)+a}-\frac{2}{a\tan^{4}\left(\frac{x}{2}\right)+2a\tan^{2}\left(\frac{x}{2}\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+a*cos(x)),x)
```

```
[Out] -4*tan(x/2)**2/(a*tan(x/2)**4 + 2*a*tan(x/2)**2 + a) - 2/(a*tan(x/2)**4 + 2
*a*tan(x/2)**2 + a)
```

Giac [A]
time = 0.41, size = 14, normalized size = 0.74

$$\frac{\cos(x)^2 - 2\cos(x)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] 1/2*(cos(x)^2 - 2*cos(x))/a
```

Mupad [B]
time = 0.25, size = 11, normalized size = 0.58

$$\frac{\cos(x)\,\left(\cos(x) - 2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + a*cos(x)),x)
```

```
[Out] (cos(x)*(cos(x) - 2))/(2*a)
```

3.3 $\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=13

$$\frac{x}{a}-\frac{\sin(x)}{a}$$

[Out] x/a-sin(x)/a

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {2761, 8}

$$\frac{x}{a}-\frac{\sin(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^2/(a + a*Cos[x]),x]

[Out] x/a - Sin[x]/a

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{a+a\cos(x)}\,dx &= -\frac{\sin(x)}{a}+\frac{\int 1\,dx}{a}\\ &= \frac{x}{a}-\frac{\sin(x)}{a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.31

$$\frac{2\left(\frac{x}{2}-\frac{\sin(x)}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + a*Cos[x]),x]
```

```
[Out] (2*(x/2 - Sin[x]/2))/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.
time = 0.06, size = 30, normalized size = 2.31

method	result	size
risch	$\frac{x}{a}-\frac{\sin(x)}{a}$	14
default	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)+1}+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$	30
norman	$\frac{\frac{x}{a}+\frac{x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{a}-\frac{2\tan\left(\frac{x}{2}\right)}{a}-\frac{2\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{a}+\frac{2x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a}}{\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)^{2}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a*(-1/2*tan(1/2*x)/(tan(1/2*x)^2+1)+1/2*arctan(tan(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(13) = 26$.
time = 0.51, size = 42, normalized size = 3.23

$$\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}-\frac{2\sin(x)}{\left(a+\frac{a\sin(x)^{2}}{(\cos(x)+1)^{2}}\right)(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(x)/(cos(x) + 1))/a - 2*sin(x)/((a + a*sin(x)^2/(cos(x) + 1)^2)
*(cos(x) + 1))
```

Fricas [A]
time = 0.37, size = 10, normalized size = 0.77

$$\frac{x-\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] (x - sin(x))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(7) = 14$.
time = 0.19, size = 46, normalized size = 3.54

$$\frac{x\tan^2\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right)+a}+\frac{x}{a\tan^2\left(\frac{x}{2}\right)+a}-\frac{2\tan\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+a*cos(x)),x)
```

```
[Out] x*tan(x/2)**2/(a*tan(x/2)**2 + a) + x/(a*tan(x/2)**2 + a) - 2*tan(x/2)/(a*t
an(x/2)**2 + a)
```

Giac [A]
time = 0.46, size = 25, normalized size = 1.92

$$\frac{x}{a}-\frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] x/a - 2*tan(1/2*x)/((tan(1/2*x)^2 + 1)*a)
```

Mupad [B]
time = 0.29, size = 10, normalized size = 0.77

$$\frac{x-\sin\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + a*cos(x)),x)
```

```
[Out] (x - sin(x))/a
```

3.4 $\int \frac{\sin(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=10

$$-\frac{\log(1+\cos(x))}{a}$$

```
[Out] -ln(cos(x)+1)/a
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2746, 31}

$$-\frac{\log(\cos(x)+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + a*Cos[x]),x]
```

```
[Out] -(Log[1 + Cos[x]]/a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a+a\cos(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\cos(x)\right)}{a}\\ &= -\frac{\log(1+\cos(x))}{a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.20

$$-\frac{2\log\left(\cos\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + a*Cos[x]),x]
```

```
[Out] (-2*Log[Cos[x/2]])/a
```

Maple [A]
time = 0.04, size = 13, normalized size = 1.30

method	result	size
derivativedivides	$-\frac{\ln(a+a\cos(x))}{a}$	13
default	$-\frac{\ln(a+a\cos(x))}{a}$	13
norman	$\frac{\ln\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{a}$	14
risch	$\frac{ix}{a}-\frac{2\ln\left(e^{ix}+1\right)}{a}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(a+a*cos(x))/a
```

Maxima [A]
time = 0.30, size = 12, normalized size = 1.20

$$-\frac{\log\left(a\cos\left(x\right)+a\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] -log(a*cos(x) + a)/a
```

Fricas [A]
time = 0.36, size = 12, normalized size = 1.20

$$-\frac{\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] -log(1/2*cos(x) + 1/2)/a
```

Sympy [A]
time = 0.05, size = 8, normalized size = 0.80

$$-\frac{\log\left(\cos\left(x\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*cos(x)),x)
```

```
[Out] -log(cos(x) + 1)/a
```

<u>**Giac**</u> [**A**]
time = 0.41, size = 10, normalized size = 1.00

$$-\frac{\log(\cos(x)+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] -log(cos(x) + 1)/a
```

<u>**Mupad**</u> [**B**]
time = 0.06, size = 10, normalized size = 1.00

$$-\frac{\ln(\cos(x)+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + a*cos(x)),x)
```

```
[Out] -log(cos(x) + 1)/a
```

3.5 $\int \frac{1}{a+a\cos(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\sin(x)}{a+a\cos(x)}$$

[Out] sin(x)/(a+a*cos(x))

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2727}

$$\frac{\sin(x)}{a\cos(x)+a}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[x])^(-1),x]

[Out] Sin[x]/(a + a*Cos[x])

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{a+a\cos(x)}\,dx = \frac{\sin(x)}{a+a\cos(x)}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 0.91

$$\frac{\tan\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[x])^(-1),x]

[Out] Tan[x/2]/a

Maple [A]
time = 0.03, size = 9, normalized size = 0.82

method	result	size
default	$\frac{\tan\left(\frac{x}{2}\right)}{a}$	9
norman	$\frac{\tan\left(\frac{x}{2}\right)}{a}$	9
risch	$\frac{2i}{(e^{ix}+1)a}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*tan(1/2*x)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 1.09

$$\frac{\sin\left(x\right)}{a(\cos\left(x\right)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] sin(x)/(a*(cos(x) + 1))
```

Fricas [A]
time = 0.35, size = 11, normalized size = 1.00

$$\frac{\sin\left(x\right)}{a\cos\left(x\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] sin(x)/(a*cos(x) + a)
```

Sympy [A]
time = 0.09, size = 5, normalized size = 0.45

$$\frac{\tan\left(\frac{x}{2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(x)),x)
```

```
[Out] tan(x/2)/a
```

Giac [A]
time = 0.42, size = 8, normalized size = 0.73

$$\frac{\tan\left(\frac{1}{2}\,x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] tan(1/2*x)/a
```

Mupad [B]
time = 0.29, size = 8, normalized size = 0.73

$$\frac{\tan\left(\frac{x}{2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a*cos(x)),x)
```

```
[Out] tan(x/2)/a
```

3.6 $\int \frac{\csc(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}(\cos(x))}{2a}+\frac{1}{2(a+a\cos(x))}$$

```
[Out] -1/2*arctanh(cos(x))/a+1/2/(a+a*cos(x))
```

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2746, 46, 212}

$$\frac{1}{2(a\cos(x)+a)}-\frac{\tanh^{-1}(\cos(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + a*Cos[x]),x]
```

```
[Out] -1/2*ArcTanh[Cos[x]]/a + 1/(2*(a + a*Cos[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{a+a\cos(x)}\,dx &= -\left(a\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^2}\,dx, x, a\cos(x)\right)\right)\\ &= -\left(a\text{Subst}\left(\int \left(\frac{1}{2a(a+x)^2}+\frac{1}{2a\,(a^2-x^2)}\right)\,dx, x, a\cos(x)\right)\right)\\ &= \frac{1}{2(a+a\cos(x))}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\cos(x)\right)\\ &= -\frac{\tanh^{-1}(\cos(x))}{2a}+\frac{1}{2(a+a\cos(x))}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 1.83

$$\frac{1-2\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{2a(1+\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + a*Cos[x]),x]
[Out] (1 - 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(2*a*(1 + Cos[x]))
```

Maple [A]
time = 0.07, size = 28, normalized size = 1.22

method	result	size
norman	$\frac{\tan^2\left(\frac{x}{2}\right)}{4a}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}$	23
default	$\frac{\frac{\ln(-1+\cos(x))}{4}+\frac{1}{2\cos(x)+2}-\frac{\ln(\cos(x)+1)}{4}}{a}$	28
risch	$\frac{e^{ix}}{\left(e^{ix}+1\right)^2 a}+\frac{\ln\left(e^{ix}-1\right)}{2a}-\frac{\ln\left(e^{ix}+1\right)}{2a}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+a*cos(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(1/4*ln(-1+cos(x))+1/2/(cos(x)+1)-1/4*ln(cos(x)+1))
```

Maxima [A]
time = 0.29, size = 31, normalized size = 1.35

$$-\frac{\log\left(\cos\left(x\right)+1\right)}{4\,a}+\frac{\log\left(\cos\left(x\right)-1\right)}{4\,a}+\frac{1}{2\left(a\cos\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(x) + 1)/a + 1/4*log(cos(x) - 1)/a + 1/2/(a*cos(x) + a)
```

Fricas [A]
time = 0.37, size = 37, normalized size = 1.61

$$-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{4\,(a\cos(x)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) - 2)/(a*cos(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc(x)}{\cos(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*cos(x)),x)
```

```
[Out] Integral(csc(x)/(cos(x) + 1), x)/a
```

Giac [A]
time = 0.48, size = 34, normalized size = 1.48

$$-\frac{\log(\cos(x)+1)}{4\,a}+\frac{\log(-\cos(x)+1)}{4\,a}+\frac{1}{2\,a(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(cos(x) + 1)/a + 1/4*log(-cos(x) + 1)/a + 1/2/(a*(cos(x) + 1))
```

Mupad [B]
time = 0.28, size = 20, normalized size = 0.87

$$\frac{1}{2\,a\,(\cos(x)+1)}-\frac{\mathrm{atanh}(\cos(x))}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + a*cos(x))),x)
```

```
[Out] 1/(2*a*(cos(x) + 1)) - atanh(cos(x))/(2*a)
```

3.7 $\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=24

$$-\frac{2\cot(x)}{3a}+\frac{\csc(x)}{3(a+a\cos(x))}$$

```
[Out] -2/3*cot(x)/a+1/3*csc(x)/(a+a*cos(x))
```

Rubi [A]

time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {2751, 3852, 8}

$$\frac{\csc(x)}{3(a\cos(x)+a)}-\frac{2\cot(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + a*Cos[x]),x]
```

```
[Out] (-2*Cot[x])/(3*a) + Csc[x]/(3*(a + a*Cos[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc^2(x)}{a+a\cos(x)}\,dx = \frac{\csc(x)}{3(a+a\cos(x))} + \frac{2\int \csc^2(x)\,dx}{3a}$$
$$= \frac{\csc(x)}{3(a+a\cos(x))} - \frac{2\text{Subst}(\int 1\,dx, x, \cot(x))}{3a}$$
$$= -\frac{2\cot(x)}{3a} + \frac{\csc(x)}{3(a+a\cos(x))}$$

Mathematica [A]
time = 0.06, size = 30, normalized size = 1.25

$$-\frac{(2\cos(x)+\cos(2x))\csc\left(\frac{x}{2}\right)\sec^3\left(\frac{x}{2}\right)}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + a*Cos[x]),x]
```

```
[Out] -1/12*((2*Cos[x] + Cos[2*x])*Csc[x/2]*Sec[x/2]^3)/a
```

Maple [A]
time = 0.07, size = 29, normalized size = 1.21

method	result	size
default	$\frac{\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}+2\tan\left(\frac{x}{2}\right)-\frac{1}{\tan\left(\frac{x}{2}\right)}}{4a}$	29
risch	$-\frac{4i(1+2\,e^{ix})}{3(e^{ix}+1)^3 a(e^{ix}-1)}$	34
norman	$\frac{-\frac{1}{4a}+\frac{\tan^2\left(\frac{x}{2}\right)}{2a}+\frac{\tan^4\left(\frac{x}{2}\right)}{12a}}{\tan\left(\frac{x}{2}\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*(1/3*tan(1/2*x)^3+2*tan(1/2*x)-1/tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.
time = 0.26, size = 41, normalized size = 1.71

$$\frac{\frac{6\,\sin(x)}{\cos(x)+1}+\frac{\sin(x)^3}{(\cos(x)+1)^3}}{12\,a} - \frac{\cos(x)+1}{4\,a\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] 1/12*(6*sin(x)/(cos(x) + 1) + sin(x)^3/(cos(x) + 1)^3)/a - 1/4*(cos(x) + 1)
/(a*sin(x))
```

Fricas [A]
time = 0.35, size = 26, normalized size = 1.08

$$-\frac{2\cos(x)^2+2\cos(x)-1}{3\,(a\cos(x)+a)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(x)^2 + 2*cos(x) - 1)/((a*cos(x) + a)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2(x)}{\cos(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+a*cos(x)),x)
```

```
[Out] Integral(csc(x)**2/(cos(x) + 1), x)/a
```

Giac [A]
time = 0.41, size = 37, normalized size = 1.54

$$\frac{a^2\tan\left(\frac{1}{2}\,x\right)^3+6\,a^2\tan\left(\frac{1}{2}\,x\right)}{12\,a^3}-\frac{1}{4\,a\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] 1/12*(a^2*tan(1/2*x)^3 + 6*a^2*tan(1/2*x))/a^3 - 1/4/(a*tan(1/2*x))
```

Mupad [B]
time = 0.32, size = 35, normalized size = 1.46

$$\frac{-8\cos\left(\frac{x}{2}\right)^4+4\cos\left(\frac{x}{2}\right)^2+1}{12\,a\cos\left(\frac{x}{2}\right)^3\,\sin\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + a*cos(x))),x)
```

```
[Out] (4*cos(x/2)^2 - 8*cos(x/2)^4 + 1)/(12*a*cos(x/2)^3*sin(x/2))
```

3.8 $\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=49

$$-\frac{3\tanh^{-1}(\cos(x))}{8a}-\frac{1}{8(a-a\cos(x))}+\frac{a}{8(a+a\cos(x))^2}+\frac{1}{4(a+a\cos(x))}$$

```
[Out] -3/8*arctanh(cos(x))/a-1/8/(a-a*cos(x))+1/8*a/(a+a*cos(x))^2+1/4/(a+a*cos(x
))
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2746, 46, 212}

$$\frac{a}{8(a\cos(x)+a)^2}-\frac{1}{8(a-a\cos(x))}+\frac{1}{4(a\cos(x)+a)}-\frac{3\tanh^{-1}(\cos(x))}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + a*Cos[x]),x]
```

```
[Out] (-3*ArcTanh[Cos[x]])/(8*a) - 1/(8*(a - a*Cos[x])) + a/(8*(a + a*Cos[x])^2)
+ 1/(4*(a + a*Cos[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\csc^3(x)}{a+a\cos(x)}\,dx = -\left(a^3\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx,x,a\cos(x)\right)\right)$$
$$= -\left(a^3\text{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2}+\frac{1}{4a^2(a+x)^3}+\frac{1}{4a^3(a+x)^2}+\frac{3}{8a^3\left(a^2-x^2\right)}\right)\,dx,x,a\right.\right.$$
$$= -\frac{1}{8(a-a\cos(x))}+\frac{a}{8(a+a\cos(x))^2}+\frac{1}{4(a+a\cos(x))}-\frac{3}{8}\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx,x,a\right.$$
$$= -\frac{3\tanh^{-1}(\cos(x))}{8a}-\frac{1}{8(a-a\cos(x))}+\frac{a}{8(a+a\cos(x))^2}+\frac{1}{4(a+a\cos(x))}$$

Mathematica [A]
time = 0.13, size = 60, normalized size = 1.22

$$\frac{4-2\cot^2\left(\frac{x}{2}\right)-12\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+\sec^2\left(\frac{x}{2}\right)}{16a(1+\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + a*Cos[x]),x]
```

```
[Out] (4 - 2*Cot[x/2]^2 - 12*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]) + Sec[x/2
]^2)/(16*a*(1 + Cos[x]))
```

Maple [A]
time = 0.10, size = 44, normalized size = 0.90

method	result	size
default	$\frac{\frac{1}{-8+8\cos(x)}+\frac{3\ln(-1+\cos(x))}{16}+\frac{1}{8(\cos(x)+1)^2}+\frac{1}{4\cos(x)+4}-\frac{3\ln(\cos(x)+1)}{16}}{a}$	44
norman	$\frac{-\frac{1}{16a}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{16a}+\frac{\tan^6\left(\frac{x}{2}\right)}{32a}}{\tan\left(\frac{x}{2}\right)^2}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8a}$	47
risch	$\frac{3\,e^{5ix}+6\,e^{4ix}-2\,e^{3ix}+6\,e^{2ix}+3\,e^{ix}}{4\left(e^{ix}+1\right)^4 a\left(e^{ix}-1\right)^2}-\frac{3\ln\left(e^{ix}+1\right)}{8a}+\frac{3\ln\left(e^{ix}-1\right)}{8a}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/8/(-1+cos(x))+3/16*ln(-1+cos(x))+1/8/(cos(x)+1)^2+1/4/(cos(x)+1)-3/1
6*ln(cos(x)+1))
```

Maxima [A]
time = 0.27, size = 58, normalized size = 1.18

$$\frac{3\,\cos\left(x\right)^2+3\,\cos\left(x\right)-2}{8\,\left(a\cos\left(x\right)^3+a\cos\left(x\right)^2-a\cos\left(x\right)-a\right)}-\frac{3\,\log\left(\cos\left(x\right)+1\right)}{16\,a}+\frac{3\,\log\left(\cos\left(x\right)-1\right)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] 1/8*(3*cos(x)^2 + 3*cos(x) - 2)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) - a) -
3/16*log(cos(x) + 1)/a + 3/16*log(cos(x) - 1)/a
```

Fricas [A]
time = 0.37, size = 83, normalized size = 1.69

$$\frac{6\,\cos\left(x\right)^{2}-3\,\left(\cos\left(x\right)^{3}+\cos\left(x\right)^{2}-\cos\left(x\right)-1\right)\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+3\,\left(\cos\left(x\right)^{3}+\cos\left(x\right)^{2}-\cos\left(x\right)-1\right)\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+6\,\cos\left(x\right)-4}{16\,\left(a\cos\left(x\right)^{3}+a\cos\left(x\right)^{2}-a\cos\left(x\right)-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/16*(6*cos(x)^2 - 3*(cos(x)^3 + cos(x)^2 - cos(x) - 1)*log(1/2*cos(x) + 1/
2) + 3*(cos(x)^3 + cos(x)^2 - cos(x) - 1)*log(-1/2*cos(x) + 1/2) + 6*cos(x)
 - 4)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^{3}\left(x\right)}{\cos\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+a*cos(x)),x)
```

```
[Out] Integral(csc(x)**3/(cos(x) + 1), x)/a
```

Giac [A]
time = 0.40, size = 52, normalized size = 1.06

$$-\frac{3\,\log\left(\cos\left(x\right)+1\right)}{16\,a}+\frac{3\,\log\left(-\cos\left(x\right)+1\right)}{16\,a}+\frac{3\,\cos\left(x\right)^{2}+3\,\cos\left(x\right)-2}{8\,a(\cos\left(x\right)+1)^{2}(\cos\left(x\right)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] -3/16*log(cos(x) + 1)/a + 3/16*log(-cos(x) + 1)/a + 1/8*(3*cos(x)^2 + 3*cos
(x) - 2)/(a*(cos(x) + 1)^2*(cos(x) - 1))
```

Mupad [B]
time = 0.29, size = 45, normalized size = 0.92

$$-\frac{\frac{3\cos(x)^{2}}{8}+\frac{3\,\cos(x)}{8}-\frac{1}{4}}{-a\cos\left(x\right)^{3}-a\cos\left(x\right)^{2}+a\,\cos\left(x\right)+a}-\frac{3\,\mathrm{atanh}(\cos\left(x\right))}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + a*cos(x))),x)
```

```
[Out] - ((3*cos(x))/8 + (3*cos(x)^2)/8 - 1/4)/(a + a*cos(x) - a*cos(x)^2 - a*cos(
x)^3) - (3*atanh(cos(x)))/(8*a)
```

3.9 $\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{4\cot(x)}{5a}-\frac{4\cot^3(x)}{15a}+\frac{\csc^3(x)}{5(a+a\cos(x))}$$

```
[Out] -4/5*cot(x)/a-4/15*cot(x)^3/a+1/5*csc(x)^3/(a+a*cos(x))
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2751, 3852}

$$-\frac{4\cot^3(x)}{15a}-\frac{4\cot(x)}{5a}+\frac{\csc^3(x)}{5(a\cos(x)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + a*Cos[x]),x]
```

```
[Out] (-4*Cot[x])/(5*a) - (4*Cot[x]^3)/(15*a) + Csc[x]^3/(5*(a + a*Cos[x]))
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(x)}{a+a\cos(x)}\,dx &= \frac{\csc^3(x)}{5(a+a\cos(x))}+\frac{4\int \csc^4(x)\,dx}{5a}\\ &= \frac{\csc^3(x)}{5(a+a\cos(x))}-\frac{4\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,\cot(x)\right)}{5a}\\ &= -\frac{4\cot(x)}{5a}-\frac{4\cot^3(x)}{15a}+\frac{\csc^3(x)}{5(a+a\cos(x))}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 38, normalized size = 1.03

$$\frac{(-6\cos(x)-2\cos(2x)+2\cos(3x)+\cos(4x))\csc^3(x)}{15a(1+\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + a*Cos[x]),x]
```

```
[Out] ((-6*Cos[x] - 2*Cos[2*x] + 2*Cos[3*x] + Cos[4*x])*Csc[x]^3)/(15*a*(1 + Cos[
x]))
```

Maple [A]
time = 0.09, size = 45, normalized size = 1.22

method	result	size
default	$\frac{\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{5}+\frac{4\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}+6\tan\left(\frac{x}{2}\right)-\frac{4}{\tan\left(\frac{x}{2}\right)}-\frac{1}{3\tan\left(\frac{x}{2}\right)^3}}{16a}$	45
risch	$\frac{16i\left(6\,e^{3ix}+2\,e^{2ix}-2\,e^{ix}-1\right)}{15\left(e^{ix}-1\right)^3 a\left(e^{ix}+1\right)^5}$	48
norman	$\frac{-\frac{1}{48a}-\frac{\tan^2\left(\frac{x}{2}\right)}{4a}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{8a}+\frac{\tan^6\left(\frac{x}{2}\right)}{12a}+\frac{\tan^8\left(\frac{x}{2}\right)}{80a}}{\tan\left(\frac{x}{2}\right)^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a*(1/5*tan(1/2*x)^5+4/3*tan(1/2*x)^3+6*tan(1/2*x)-4/tan(1/2*x)-1/3/tan
(1/2*x)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(31) = 62$.
time = 0.30, size = 70, normalized size = 1.89

$$\frac{\frac{90\sin(x)}{\cos(x)+1}+\frac{20\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^5}{(\cos(x)+1)^5}}{240\,a}-\frac{\left(\frac{12\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)^3}{48\,a\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] 1/240*(90*sin(x)/(cos(x) + 1) + 20*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^5/(co
s(x) + 1)^5)/a - 1/48*(12*sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)^3/(a*si
n(x)^3)
```

Fricas [A]
time = 0.35, size = 53, normalized size = 1.43

$$-\frac{8\cos\left(x\right)^{4}+8\cos\left(x\right)^{3}-12\cos\left(x\right)^{2}-12\cos\left(x\right)+3}{15\left(a\cos\left(x\right)^{3}+a\cos\left(x\right)^{2}-a\cos\left(x\right)-a\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(x)^4 + 8*cos(x)^3 - 12*cos(x)^2 - 12*cos(x) + 3)/((a*cos(x)^3
+ a*cos(x)^2 - a*cos(x) - a)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^{4}\left(x\right)}{\cos\left(x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+a*cos(x)),x)
```

```
[Out] Integral(csc(x)**4/(cos(x) + 1), x)/a
```

Giac [A]
time = 0.56, size = 59, normalized size = 1.59

$$-\frac{12\tan\left(\frac{1}{2}x\right)^{2}+1}{48\,a\tan\left(\frac{1}{2}x\right)^{3}}+\frac{3\,a^{4}\tan\left(\frac{1}{2}x\right)^{5}+20\,a^{4}\tan\left(\frac{1}{2}x\right)^{3}+90\,a^{4}\tan\left(\frac{1}{2}x\right)}{240\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] -1/48*(12*tan(1/2*x)^2 + 1)/(a*tan(1/2*x)^3) + 1/240*(3*a^4*tan(1/2*x)^5 +
20*a^4*tan(1/2*x)^3 + 90*a^4*tan(1/2*x))/a^5
```

Mupad [B]
time = 0.37, size = 45, normalized size = 1.22

$$\frac{3\tan\left(\frac{x}{2}\right)^{8}+20\tan\left(\frac{x}{2}\right)^{6}+90\tan\left(\frac{x}{2}\right)^{4}-60\tan\left(\frac{x}{2}\right)^{2}-5}{240\,a\tan\left(\frac{x}{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + a*cos(x))),x)
```

```
[Out] (90*tan(x/2)^4 - 60*tan(x/2)^2 + 20*tan(x/2)^6 + 3*tan(x/2)^8 - 5)/(240*a*t
an(x/2)^3)
```

3.10 $\int \frac{\sin(2x)}{1+\cos(2x)}\,dx$

Optimal. Leaf size=5

$$-\log(\cos(x))$$

```
[Out] -ln(cos(x))
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10. time = 0.01, antiderivative size = 11, normalized size of antiderivative = 2.20, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2746, 31}

$$-\frac{1}{2}\log(\cos(2x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(1 + Cos[2*x]),x]
```

```
[Out] -1/2*Log[1 + Cos[2*x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{1+\cos(2x)}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(2x)\right)\right)\\ &= -\frac{1}{2}\log(1+\cos(2x))\end{aligned}$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(1 + Cos[2*x]),x]
```

```
[Out] -Log[Cos[x]]
```

Maple [A]
time = 0.04, size = 10, normalized size = 2.00

method	result	size
derivativedivides	$-\frac{\ln(\cos(2x)+1)}{2}$	10
default	$-\frac{\ln(\cos(2x)+1)}{2}$	10
norman	$\frac{\ln(1+\tan^2(x))}{2}$	10
risch	$ix - \ln\left(e^{2ix}+1\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(cos(2*x)+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(cos(2*x)+1)
```

Maxima [A]
time = 0.31, size = 9, normalized size = 1.80

$$-\frac{1}{2}\,\log\left(\cos\left(2\,x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(1+cos(2*x)),x, algorithm="maxima")
```

```
[Out] -1/2*log(cos(2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.
time = 0.38, size = 11, normalized size = 2.20

$$-\frac{1}{2}\,\log\left(\frac{1}{2}\,\cos\left(2\,x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(1+cos(2*x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*cos(2*x) + 1/2)
```

Sympy [A]
time = 0.04, size = 10, normalized size = 2.00

$$-\frac{\log\left(\cos\left(2x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(1+cos(2*x)),x)
[Out] -log(cos(2*x) + 1)/2
```

Giac [A]
time = 0.42, size = 9, normalized size = 1.80

$$-\frac{1}{2}\log\left(\cos\left(2\,x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(1+cos(2*x)),x, algorithm="giac")
[Out] -1/2*log(cos(2*x) + 1)
```

Mupad [B]
time = 0.26, size = 7, normalized size = 1.40

$$-\frac{\ln\left(\cos\left(x\right)^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(cos(2*x) + 1),x)
[Out] -log(cos(x)^2)/2
```

3.11 $\int \frac{\sin(2x)}{1-\cos(2x)}\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] ln(sin(x))
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 13 vs. $2(3) = 6$.
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 4.33, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2746, 31}

$$\frac{1}{2}\log(1-\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(1 - Cos[2*x]),x]
```

```
[Out] Log[1 - Cos[2*x]]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{1-\cos(2x)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, -\cos(2x)\right)\\ &= \frac{1}{2}\log(1-\cos(2x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(1 - Cos[2*x]),x]
```

```
[Out] Log[Sin[x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.
time = 0.04, size = 12, normalized size = 4.00

method	result	size
derivativedivides	$\frac{\ln(1-\cos(2x))}{2}$	12
default	$\frac{\ln(1-\cos(2x))}{2}$	12
norman	$-\frac{\ln(1+\tan^2(x))}{2}+\ln(\tan(x))$	14
risch	$-ix+\ln(\mathrm{e}^{2ix}-1)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(1-cos(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(1-cos(2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 9 vs. $2(3) = 6$.
time = 0.27, size = 9, normalized size = 3.00

$$\frac{1}{2}\log\left(\cos\left(2x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(1-cos(2*x)),x, algorithm="maxima")
```

```
[Out] 1/2*log(cos(2*x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.
time = 0.37, size = 11, normalized size = 3.67

$$\frac{1}{2}\log\left(-\frac{1}{2}\cos\left(2x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(1-cos(2*x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 8 vs. $2(3) = 6$.
time = 0.04, size = 8, normalized size = 2.67

$$\frac{\log\left(\cos\left(2x\right)-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(1-cos(2*x)),x)
```

```
[Out] log(cos(2*x) - 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(3) = 6.
time = 0.42, size = 11, normalized size = 3.67

$$\frac{1}{2}\log\left(-\cos\left(2x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(1-cos(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*log(-cos(2*x) + 1)
```

Mupad [B]
time = 0.08, size = 9, normalized size = 3.00

$$\frac{\ln\left(-\sin\left(x\right)^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(2*x)/(cos(2*x) - 1),x)
```

```
[Out] log(-sin(x)^2)/2
```

3.12 $\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx$

Optimal. Leaf size=6

$$\frac{1}{1+\cos(x)}$$

[Out] 1/(cos(x)+1)

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2746, 32}

$$\frac{1}{\cos(x)+1}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(1 + Cos[x])^2,x]

[Out] (1 + Cos[x])^(-1)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{(1+\cos(x))^2}\,dx &= -\text{Subst}\left(\int \frac{1}{(1+x)^2}\,dx, x, \cos(x)\right)\\ &= \frac{1}{1+\cos(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 2.00

$$\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(1 + Cos[x])^2,x]
```

```
[Out] Sec[x/2]^2/2
```

Maple [A]
time = 0.04, size = 7, normalized size = 1.17

method	result	size
derivativedivides	$\frac{1}{\cos(x)+1}$	7
default	$\frac{1}{\cos(x)+1}$	7
norman	$\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}$	9
risch	$\frac{2\,\mathrm{e}^{ix}}{\left(\mathrm{e}^{ix}+1\right)^2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(cos(x)+1)
```

Maxima [A]
time = 0.26, size = 6, normalized size = 1.00

$$\frac{1}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x))^2,x, algorithm="maxima")
```

```
[Out] 1/(cos(x) + 1)
```

Fricas [A]
time = 0.35, size = 6, normalized size = 1.00

$$\frac{1}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x))^2,x, algorithm="fricas")
```

```
[Out] 1/(cos(x) + 1)
```

Sympy [A]
time = 0.15, size = 5, normalized size = 0.83

$$\frac{1}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x))**2,x)
```

```
[Out] 1/(cos(x) + 1)
```

Giac [A]
time = 0.40, size = 6, normalized size = 1.00

$$\frac{1}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x))^2,x, algorithm="giac")
```

```
[Out] 1/(cos(x) + 1)
```

Mupad [B]
time = 0.25, size = 6, normalized size = 1.00

$$\frac{1}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x) + 1)^2,x)
```

```
[Out] 1/(cos(x) + 1)
```

3.13 $\int \frac{\sin(x)}{(1-\cos(x))^2} \, dx$

Optimal. Leaf size=10

$$-\frac{1}{1-\cos(x)}$$

[Out] -1/(1-cos(x))

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2746, 32}

$$-\frac{1}{1-\cos(x)}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(1 - Cos[x])^2,x]

[Out] -(1 - Cos[x])^(-1)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned} \int \frac{\sin(x)}{(1-\cos(x))^2} \, dx &= \text{Subst}\left(\int \frac{1}{(1+x)^2} \, dx, x, -\cos(x)\right) \\ &= -\frac{1}{1-\cos(x)} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.20

$$-\frac{1}{2} \csc^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(1 - Cos[x])^2,x]
[Out] -1/2*Csc[x/2]^2
```

Maple [A]
time = 0.05, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$-\frac{1}{1-\cos(x)}$	11
default	$-\frac{1}{1-\cos(x)}$	11
risch	$\frac{2\,\mathrm{e}^{ix}}{(\mathrm{e}^{ix}-1)^2}$	17
norman	$\frac{-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}-\frac{\tan\left(\frac{x}{2}\right)}{2}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan\left(\frac{x}{2}\right)^3}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1-cos(x))^2,x,method=_RETURNVERBOSE)
[Out] -1/(1-cos(x))
```

Maxima [A]
time = 0.29, size = 6, normalized size = 0.60

$$\frac{1}{\cos(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1-cos(x))^2,x, algorithm="maxima")
[Out] 1/(cos(x) - 1)
```

Fricas [A]
time = 0.38, size = 6, normalized size = 0.60

$$\frac{1}{\cos(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1-cos(x))^2,x, algorithm="fricas")
[Out] 1/(cos(x) - 1)
```

Sympy [A]
time = 0.16, size = 5, normalized size = 0.50

$$\frac{1}{\cos(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1-cos(x))**2,x)
```

```
[Out] 1/(cos(x) - 1)
```

Giac [A]
time = 0.45, size = 6, normalized size = 0.60

$$\frac{1}{\cos(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1-cos(x))^2,x, algorithm="giac")
```

```
[Out] 1/(cos(x) - 1)
```

Mupad [B]
time = 0.25, size = 6, normalized size = 0.60

$$\frac{1}{\cos(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x) - 1)^2,x)
```

```
[Out] 1/(cos(x) - 1)
```

3.14 $\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx$

Optimal. Leaf size=14

$$-x+\frac{2\sin(x)}{1+\cos(x)}$$

```
[Out] -x+2*sin(x)/(cos(x)+1)
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2759, 8}

$$\frac{2\sin(x)}{\cos(x)+1}-x$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(1 + Cos[x])^2,x]
```

```
[Out] -x + (2*Sin[x])/(1 + Cos[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx &= \frac{2\sin(x)}{1+\cos(x)}-\int 1\,dx\\ &= -x+\frac{2\sin(x)}{1+\cos(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.29

$$-2\text{ArcTan}\left(\tan\left(\frac{x}{2}\right)\right)+2\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(1 + Cos[x])^2,x]
```

```
[Out] -2*ArcTan[Tan[x/2]] + 2*Tan[x/2]
```

Maple [A]
time = 0.05, size = 15, normalized size = 1.07

method	result	size
default	$2\tan\left(\frac{x}{2}\right)-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	15
risch	$-x+\frac{4i}{e^{ix}+1}$	17
norman	$\frac{-x+4\left(\tan^{3}\left(\frac{x}{2}\right)\right)+2\left(\tan^{5}\left(\frac{x}{2}\right)\right)-2x\left(\tan^{2}\left(\frac{x}{2}\right)\right)-x\left(\tan^{4}\left(\frac{x}{2}\right)\right)+2\tan\left(\frac{x}{2}\right)}{\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)^{2}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*tan(1/2*x)-2*arctan(tan(1/2*x))
```

Maxima [A]
time = 0.48, size = 23, normalized size = 1.64

$$\frac{2\sin\left(x\right)}{\cos\left(x\right)+1}-2\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+cos(x))^2,x, algorithm="maxima")
```

```
[Out] 2*sin(x)/(cos(x) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 0.35, size = 18, normalized size = 1.29

$$-\frac{x\cos\left(x\right)+x-2\sin\left(x\right)}{\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+cos(x))^2,x, algorithm="fricas")
```

```
[Out] -(x*cos(x) + x - 2*sin(x))/(cos(x) + 1)
```

Sympy [A]
time = 0.21, size = 7, normalized size = 0.50

$$-x+2\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(1+cos(x))**2,x)
```

```
[Out] -x + 2*tan(x/2)
```

Giac [A]
time = 0.41, size = 10, normalized size = 0.71

$$-x + 2 \tan\left(\frac{1}{2} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+cos(x))^2,x, algorithm="giac")
```

```
[Out] -x + 2*tan(1/2*x)
```

Mupad [B]
time = 0.30, size = 10, normalized size = 0.71

$$2 \tan\left(\frac{x}{2}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(cos(x) + 1)^2,x)
```

```
[Out] 2*tan(x/2) - x
```

3.15 $\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx$

Optimal. Leaf size=16

$$-x-\frac{2\sin(x)}{1-\cos(x)}$$

```
[Out] -x-2*sin(x)/(1-cos(x))
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2759, 8}

$$-x-\frac{2\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(1 - Cos[x])^2,x]
```

```
[Out] -x - (2*Sin[x])/(1 - Cos[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx &= -\frac{2\sin(x)}{1-\cos(x)}-\int 1\,dx\\ &= -x-\frac{2\sin(x)}{1-\cos(x)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.01, size = 26, normalized size = 1.62

$$-2\cot\left(\frac{x}{2}\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(1 - Cos[x])^2,x]
```

```
[Out] -2*Cot[x/2]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x/2]^2]
```

Maple [A]
time = 0.08, size = 17, normalized size = 1.06

method	result	size
default	$-\frac{2}{\tan(\frac{x}{2})}-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	17
risch	$-x-\frac{4i}{e^{ix}-1}$	17
norman	$\frac{-2(\tan^2(\frac{x}{2}))-4(\tan^4(\frac{x}{2}))-2(\tan^6(\frac{x}{2}))-x(\tan^3(\frac{x}{2}))-2x(\tan^5(\frac{x}{2}))-x(\tan^7(\frac{x}{2}))}{(\tan^2(\frac{x}{2})+1)^2\tan(\frac{x}{2})^3}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(1-cos(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/tan(1/2*x)-2*arctan(tan(1/2*x))
```

Maxima [A]
time = 0.53, size = 23, normalized size = 1.44

$$-\frac{2\,(\cos(x)+1)}{\sin(x)}-2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1-cos(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 0.37, size = 16, normalized size = 1.00

$$-\frac{x\sin(x)+2\,\cos(x)+2}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1-cos(x))^2,x, algorithm="fricas")
```

```
[Out] -(x*sin(x) + 2*cos(x) + 2)/sin(x)
```

Sympy [A]
time = 0.37, size = 8, normalized size = 0.50

$$-x-\frac{2}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(1-cos(x))**2,x)
```

```
[Out] -x - 2/tan(x/2)
```

Giac [A]
time = 0.42, size = 12, normalized size = 0.75

$$-x-\frac{2}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1-cos(x))^2,x, algorithm="giac")
```

```
[Out] -x - 2/tan(1/2*x)
```

Mupad [B]
time = 0.31, size = 10, normalized size = 0.62

$$-x-2\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(cos(x) - 1)^2,x)
```

```
[Out] - x - 2*cot(x/2)
```

3.16 $\int \frac{\sin^3(x)}{(1+\cos(x))^2} \, dx$

Optimal. Leaf size=10

$$\cos(x) - 2\log(1 + \cos(x))$$

```
[Out] cos(x)-2*ln(cos(x)+1)
```

Rubi [A]
time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2746, 45}

$$\cos(x) - 2\log(\cos(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(1 + Cos[x])^2,x]
```

```
[Out] Cos[x] - 2*Log[1 + Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{(1+\cos(x))^2} \, dx &= -\text{Subst}\left(\int \frac{1-x}{1+x} \, dx, x, \cos(x)\right) \\
&= -\text{Subst}\left(\int \left(-1+\frac{2}{1+x}\right) \, dx, x, \cos(x)\right) \\
&= \cos(x) - 2\log(1+\cos(x))
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 13, normalized size = 1.30

$$-1+\cos(x)-4\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(1 + Cos[x])^2,x]
```

```
[Out] -1 + Cos[x] - 4*Log[Cos[x/2]]
```

Maple [A]
time = 0.06, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\cos(x)-2\ln(\cos(x)+1)$	11
default	$\cos(x)-2\ln(\cos(x)+1)$	11
risch	$2ix+\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}-4\ln(e^{ix}+1)$	30
norman	$\frac{2(\tan^4(\frac{x}{2}))+4(\tan^2(\frac{x}{2}))+2}{(\tan^2(\frac{x}{2})+1)^3}+2\ln\left(\tan^2\left(\frac{x}{2}\right)+1\right)$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(cos(x)+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] cos(x)-2*ln(cos(x)+1)
```

Maxima [A]
time = 0.27, size = 10, normalized size = 1.00

$$\cos(x)-2\log(\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(1+cos(x))^2,x, algorithm="maxima")
```

```
[Out] cos(x) - 2*log(cos(x) + 1)
```

Fricas [A]
time = 0.40, size = 12, normalized size = 1.20

$$\cos(x)-2\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(1+cos(x))^2,x, algorithm="fricas")
```

```
[Out] cos(x) - 2*log(1/2*cos(x) + 1/2)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.
time = 0.21, size = 58, normalized size = 5.80

$$-\frac{2\log\left(\cos\left(x\right)+1\right)\cos\left(x\right)}{\cos\left(x\right)+1}-\frac{2\log\left(\cos\left(x\right)+1\right)}{\cos\left(x\right)+1}+\frac{\sin^{2}\left(x\right)}{\cos\left(x\right)+1}+\frac{2\cos^{2}\left(x\right)}{\cos\left(x\right)+1}-\frac{2}{\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(1+cos(x))**2,x)
```

```
[Out] -2*log(cos(x) + 1)*cos(x)/(cos(x) + 1) - 2*log(cos(x) + 1)/(cos(x) + 1) + s
in(x)**2/(cos(x) + 1) + 2*cos(x)**2/(cos(x) + 1) - 2/(cos(x) + 1)
```

Giac [**A**]
time = 0.41, size = 10, normalized size = 1.00

$$\cos\left(x\right) - 2\,\log\left(\cos\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(1+cos(x))^2,x, algorithm="giac")
```

```
[Out] cos(x) - 2*log(cos(x) + 1)
```

Mupad [**B**]
time = 0.25, size = 10, normalized size = 1.00

$$\cos\left(x\right) - 2\,\ln\left(\cos\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(cos(x) + 1)^2,x)
```

```
[Out] cos(x) - 2*log(cos(x) + 1)
```

3.17 $\int \frac{\sin^3(x)}{(1-\cos(x))^2} dx$

Optimal. Leaf size=12

$$\cos(x) + 2\log(1 - \cos(x))$$

```
[Out] cos(x)+2*ln(1-cos(x))
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2746, 45}

$$\cos(x) + 2\log(1 - \cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(1 - Cos[x])^2,x]
```

```
[Out] Cos[x] + 2*Log[1 - Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{(1-\cos(x))^2} dx &= \text{Subst}\left(\int \frac{1-x}{1+x} dx, x, -\cos(x)\right) \\
&= \text{Subst}\left(\int \left(-1+\frac{2}{1+x}\right) dx, x, -\cos(x)\right) \\
&= \cos(x) + 2\log(1-\cos(x))
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 13, normalized size = 1.08

$$-1+\cos(x)+4\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(1 - Cos[x])^2,x]
[Out] -1 + Cos[x] + 4*Log[Sin[x/2]]
```

Maple [A]
time = 0.08, size = 11, normalized size = 0.92

method	result	size
derivativedivides	$\cos(x)+2\ln(-1+\cos(x))$	11
default	$\cos(x)+2\ln(-1+\cos(x))$	11
risch	$-2ix+\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+4\ln(e^{ix}-1)$	30
norman	$\frac{-2\left(\tan^7\left(\frac{x}{2}\right)\right)-\frac{4\left(\tan^9\left(\frac{x}{2}\right)\right)}{3}+\frac{2\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3\tan\left(\frac{x}{2}\right)^3}+4\ln\left(\tan\left(\frac{x}{2}\right)\right)-2\ln\left(\tan^2\left(\frac{x}{2}\right)+1\right)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(1-cos(x))^2,x,method=_RETURNVERBOSE)
[Out] cos(x)+2*ln(-1+cos(x))
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.83

$$\cos(x)+2\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(1-cos(x))^2,x, algorithm="maxima")
[Out] cos(x) + 2*log(cos(x) - 1)
```

Fricas [A]
time = 0.37, size = 12, normalized size = 1.00

$$\cos(x)+2\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(1-cos(x))^2,x, algorithm="fricas")
```

```
[Out] cos(x) + 2*log(-1/2*cos(x) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.
time = 0.20, size = 58, normalized size = 4.83

$$\frac{2\log\left(\cos\left(x\right)-1\right)\cos\left(x\right)}{\cos\left(x\right)-1}-\frac{2\log\left(\cos\left(x\right)-1\right)}{\cos\left(x\right)-1}+\frac{\sin^{2}\left(x\right)}{\cos\left(x\right)-1}+\frac{2\cos^{2}\left(x\right)}{\cos\left(x\right)-1}-\frac{2}{\cos\left(x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(1-cos(x))**2,x)
```

```
[Out] 2*log(cos(x) - 1)*cos(x)/(cos(x) - 1) - 2*log(cos(x) - 1)/(cos(x) - 1) + si
n(x)**2/(cos(x) - 1) + 2*cos(x)**2/(cos(x) - 1) - 2/(cos(x) - 1)
```

Giac [A]
time = 0.39, size = 12, normalized size = 1.00

$$\cos\left(x\right)+2\,\log\left(-\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(1-cos(x))^2,x, algorithm="giac")
```

```
[Out] cos(x) + 2*log(-cos(x) + 1)
```

Mupad [B]
time = 0.04, size = 10, normalized size = 0.83

$$2\,\ln\left(\cos\left(x\right)-1\right)+\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(cos(x) - 1)^2,x)
```

```
[Out] 2*log(cos(x) - 1) + cos(x)
```

3.18 $\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2(1+\cos(x))^2}$$

```
[Out] 1/2/(cos(x)+1)^2
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2746, 32}

$$\frac{1}{2(\cos(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(1 + Cos[x])^3,x]
```

```
[Out] 1/(2*(1 + Cos[x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{(1+\cos(x))^3}\,dx &= -\text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, \cos(x)\right)\\ &= \frac{1}{2(1+\cos(x))^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.20

$$\frac{1}{8}\sec^4\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(1 + Cos[x])^3,x]
[Out] Sec[x/2]^4/8
```

Maple [A]
time = 0.04, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$\frac{1}{2(\cos(x)+1)^2}$	9
default	$\frac{1}{2(\cos(x)+1)^2}$	9
risch	$\frac{2\,\mathrm{e}^{2ix}}{(\mathrm{e}^{ix}+1)^4}$	17
norman	$\frac{(\tan^2(\frac{x}{2}))}{4}+\frac{(\tan^4(\frac{x}{2}))}{8}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)+1)^3,x,method=_RETURNVERBOSE)
[Out] 1/2/(cos(x)+1)^2
```

Maxima [A]
time = 0.32, size = 8, normalized size = 0.80

$$\frac{1}{2\,(\cos{(x)}+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x))^3,x, algorithm="maxima")
[Out] 1/2/(cos(x) + 1)^2
```

Fricas [A]
time = 0.36, size = 14, normalized size = 1.40

$$\frac{1}{2\,\left(\cos{(x)}^2+2\,\cos{(x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x))^3,x, algorithm="fricas")
[Out] 1/2/(cos(x)^2 + 2*cos(x) + 1)
```

Sympy [A]
time = 0.26, size = 14, normalized size = 1.40

$$\frac{1}{2\cos^2{(x)}+4\cos{(x)}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x))**3,x)
```

```
[Out] 1/(2*cos(x)**2 + 4*cos(x) + 2)
```

Giac [A]
time = 0.49, size = 8, normalized size = 0.80

$$\frac{1}{2\left(\cos\left(x\right)+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x))^3,x, algorithm="giac")
```

```
[Out] 1/2/(cos(x) + 1)^2
```

Mupad [B]
time = 0.04, size = 8, normalized size = 0.80

$$\frac{1}{2\left(\cos\left(x\right)+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x) + 1)^3,x)
```

```
[Out] 1/(2*(cos(x) + 1)^2)
```

3.19 $\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{2(1-\cos(x))^2}$$

```
[Out] -1/2/(1-cos(x))^2
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2746, 32}

$$-\frac{1}{2(1-\cos(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(1 - Cos[x])^3,x]
```

```
[Out] -1/2*1/(1 - Cos[x])^2
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{(1-\cos(x))^3}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, -\cos(x)\right)\\ &= -\frac{1}{2(1-\cos(x))^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\frac{1}{8}\csc^4\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(1 - Cos[x])^3,x]
[Out] -1/8*Csc[x/2]^4
```

Maple [A]
time = 0.05, size = 11, normalized size = 0.92

method	result	size
derivativedivides	$-\frac{1}{2(1-\cos(x))^2}$	11
default	$-\frac{1}{2(1-\cos(x))^2}$	11
risch	$-\frac{2\,\mathrm{e}^{2ix}}{(\mathrm{e}^{ix}-1)^4}$	17
norman	$\frac{-\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{3\left(\tan^3\left(\frac{x}{2}\right)\right)}{8}-\frac{\tan\left(\frac{x}{2}\right)}{8}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan\left(\frac{x}{2}\right)^5}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1-cos(x))^3,x,method=_RETURNVERBOSE)
[Out] -1/2/(1-cos(x))^2
```

Maxima [A]
time = 0.30, size = 8, normalized size = 0.67

$$-\frac{1}{2\left(\cos\left(x\right)-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1-cos(x))^3,x, algorithm="maxima")
[Out] -1/2/(cos(x) - 1)^2
```

Fricas [A]
time = 0.35, size = 14, normalized size = 1.17

$$-\frac{1}{2\left(\cos\left(x\right)^2-2\,\cos\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1-cos(x))^3,x, algorithm="fricas")
[Out] -1/2/(cos(x)^2 - 2*cos(x) + 1)
```

Sympy [A]
time = 0.24, size = 15, normalized size = 1.25

$$-\frac{1}{2\cos^2\left(x\right)-4\cos\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1-cos(x))**3,x)
```

```
[Out] -1/(2*cos(x)**2 - 4*cos(x) + 2)
```

Giac [A]
time = 0.45, size = 8, normalized size = 0.67

$$-\frac{1}{2\left(\cos\left(x\right)-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1-cos(x))^3,x, algorithm="giac")
```

```
[Out] -1/2/(cos(x) - 1)^2
```

Mupad [B]
time = 0.04, size = 8, normalized size = 0.67

$$-\frac{1}{2\left(\cos\left(x\right)-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(x)/(cos(x) - 1)^3,x)
```

```
[Out] -1/(2*(cos(x) - 1)^2)
```

3.20 $\int \frac{\sin^2(x)}{(1+\cos(x))^3} \, dx$

Optimal. Leaf size=14

$$\frac{\sin^3(x)}{3(1+\cos(x))^3}$$

```
[Out] 1/3*sin(x)^3/(cos(x)+1)^3
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2750}

$$\frac{\sin^3(x)}{3(\cos(x)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(1 + Cos[x])^3,x]
```

```
[Out] Sin[x]^3/(3*(1 + Cos[x])^3)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^2(x)}{(1+\cos(x))^3} \, dx = \frac{\sin^3(x)}{3(1+\cos(x))^3}$$

Mathematica [A]
time = 0.03, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan^3\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(1 + Cos[x])^3,x]
```

```
[Out] Tan[x/2]^3/3
```

Maple [A]
time = 0.06, size = 9, normalized size = 0.64

method	result	size
default	$\frac{\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3}$	9
risch	$-\frac{2i\left(3\,\mathrm{e}^{2ix}+1\right)}{3\left(\mathrm{e}^{ix}+1\right)^{3}}$	22
norman	$\frac{\frac{\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3}+\frac{2\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{3}+\frac{\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{3}}{\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)^{2}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(cos(x)+1)^3,x,method=_RETURNVERBOSE)
[Out] 1/3*tan(1/2*x)^3
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.86

$$\frac{\sin\left(x\right)^{3}}{3\left(\cos\left(x\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+cos(x))^3,x, algorithm="maxima")
[Out] 1/3*sin(x)^3/(cos(x) + 1)^3
```

Fricas [A]
time = 0.35, size = 20, normalized size = 1.43

$$-\frac{\left(\cos\left(x\right)-1\right)\sin\left(x\right)}{3\left(\cos\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+cos(x))^3,x, algorithm="fricas")
[Out] -1/3*(cos(x) - 1)*sin(x)/(cos(x)^2 + 2*cos(x) + 1)
```

Sympy [A]
time = 0.34, size = 7, normalized size = 0.50

$$\frac{\tan^{3}\left(\frac{x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(1+cos(x))**3,x)
```

```
[Out] tan(x/2)**3/3
```

Giac [A]
time = 0.42, size = 8, normalized size = 0.57

$$\frac{1}{3}\tan\left(\frac{1}{2}x\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+cos(x))^3,x, algorithm="giac")
```

```
[Out] 1/3*tan(1/2*x)^3
```

Mupad [B]
time = 0.26, size = 8, normalized size = 0.57

$$\frac{\tan\left(\frac{x}{2}\right)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(cos(x) + 1)^3,x)
```

```
[Out] tan(x/2)^3/3
```

3.21 $\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx$

Optimal. Leaf size=16

$$-\frac{\sin^3(x)}{3(1-\cos(x))^3}$$

```
[Out] -1/3*sin(x)^3/(1-cos(x))^3
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{2750\}$

$$-\frac{\sin^3(x)}{3(1-\cos(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(1 - Cos[x])^3,x]
```

```
[Out] -1/3*Sin[x]^3/(1 - Cos[x])^3
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^2(x)}{(1-\cos(x))^3}\,dx = -\frac{\sin^3(x)}{3(1-\cos(x))^3}$$

Mathematica [A]
time = 0.04, size = 12, normalized size = 0.75

$$-\frac{1}{3}\cot^3\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(1 - Cos[x])^3,x]
```

```
[Out] -1/3*Cot[x/2]^3
```

Maple [A]
time = 0.09, size = 9, normalized size = 0.56

method	result	size
default	$-\frac{1}{3\tan\left(\frac{x}{2}\right)^{3}}$	9
risch	$\frac{2i\left(3\,\mathrm{e}^{2ix}+1\right)}{3\left(\mathrm{e}^{ix}-1\right)^{3}}$	22
norman	$\frac{-\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3}-\frac{2\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{3}-\frac{\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{3}}{\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)^{2}\tan\left(\frac{x}{2}\right)^{5}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(1-cos(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/tan(1/2*x)^3
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.75

$$-\frac{\left(\cos\left(x\right)+1\right)^{3}}{3\,\sin\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1-cos(x))^3,x, algorithm="maxima")
```

```
[Out] -1/3*(cos(x) + 1)^3/sin(x)^3
```

Fricas [A]
time = 0.34, size = 22, normalized size = 1.38

$$\frac{\cos\left(x\right)^{2}+2\,\cos\left(x\right)+1}{3\left(\cos\left(x\right)-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1-cos(x))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2*cos(x) + 1)/((cos(x) - 1)*sin(x))
```

Sympy [A]
time = 0.54, size = 10, normalized size = 0.62

$$-\frac{1}{3\tan^{3}\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(1-cos(x))**3,x)
```

```
[Out] -1/(3*tan(x/2)**3)
```

Giac [A]
time = 0.52, size = 8, normalized size = 0.50

$$-\frac{1}{3\,\tan\left(\frac{1}{2}\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1-cos(x))^3,x, algorithm="giac")
```

```
[Out] -1/3/tan(1/2*x)^3
```

Mupad [B]
time = 0.35, size = 8, normalized size = 0.50

$$-\frac{\cot\left(\frac{x}{2}\right)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(x)^2/(cos(x) - 1)^3,x)
```

```
[Out] -cot(x/2)^3/3
```

3.22 $\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx$

Optimal. Leaf size=14

$$\frac{2}{1+\cos(x)}+\log(1+\cos(x))$$

```
[Out] 2/(cos(x)+1)+ln(cos(x)+1)
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2746, 45}

$$\frac{2}{\cos(x)+1}+\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(1 + Cos[x])^3,x]
```

```
[Out] 2/(1 + Cos[x]) + Log[1 + Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx &= -\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx,x,\cos(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)\,dx,x,\cos(x)\right)\\
&= \frac{2}{1+\cos(x)}+\log(1+\cos(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.29

$$2\log\left(\cos\left(\frac{x}{2}\right)\right)+\tan^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(1 + Cos[x])^3,x]
```

```
[Out] 2*Log[Cos[x/2]] + Tan[x/2]^2
```

Maple [A]
time = 0.07, size = 15, normalized size = 1.07

method	result	size
derivativedivides	$\frac{2}{\cos(x)+1}+\ln\left(\cos\left(x\right)+1\right)$	15
default	$\frac{2}{\cos(x)+1}+\ln\left(\cos\left(x\right)+1\right)$	15
risch	$-ix+\frac{4\,\mathrm{e}^{ix}}{\left(\mathrm{e}^{ix}+1\right)^{2}}+2\ln\left(\mathrm{e}^{ix}+1\right)$	32
norman	$\frac{\tan^{8}\left(\frac{x}{2}\right)-2\left(\tan^{2}\left(\frac{x}{2}\right)\right)+2\left(\tan^{6}\left(\frac{x}{2}\right)\right)-1}{\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)^{3}}-\ln\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(cos(x)+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(cos(x)+1)+ln(cos(x)+1)
```

Maxima [A]
time = 0.26, size = 14, normalized size = 1.00

$$\frac{2}{\cos\left(x\right)+1}+\log\left(\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(1+cos(x))^3,x, algorithm="maxima")
```

```
[Out] 2/(cos(x) + 1) + log(cos(x) + 1)
```

Fricas [A]
time = 0.36, size = 21, normalized size = 1.50

$$\frac{\left(\cos\left(x\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+2}{\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(1+cos(x))^3,x, algorithm="fricas")
```

```
[Out] ((cos(x) + 1)*log(1/2*cos(x) + 1/2) + 2)/(cos(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(12) = 24$.
time = 0.27, size = 126, normalized size = 9.00

$$\frac{2\log(\cos(x)+1)\cos^2(x)}{2\cos^2(x)+4\cos(x)+2} + \frac{4\log(\cos(x)+1)\cos(x)}{2\cos^2(x)+4\cos(x)+2} + \frac{2\log(\cos(x)+1)}{2\cos^2(x)+4\cos(x)+2} + \frac{\sin^2(x)}{2\cos^2(x)+4\cos(x)+2} + \frac{2\cos(x)}{2\cos^2(x)+4\cos(x)+2} + \frac{2}{2\cos^2(x)+4\cos(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(1+cos(x))**3,x)
```

```
[Out] 2*log(cos(x) + 1)*cos(x)**2/(2*cos(x)**2 + 4*cos(x) + 2) + 4*log(cos(x) + 1
)*cos(x)/(2*cos(x)**2 + 4*cos(x) + 2) + 2*log(cos(x) + 1)/(2*cos(x)**2 + 4*
cos(x) + 2) + sin(x)**2/(2*cos(x)**2 + 4*cos(x) + 2) + 2*cos(x)/(2*cos(x)**
2 + 4*cos(x) + 2) + 2/(2*cos(x)**2 + 4*cos(x) + 2)
```

Giac [A]
time = 0.44, size = 14, normalized size = 1.00

$$\frac{2}{\cos(x)+1} + \log(\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(1+cos(x))^3,x, algorithm="giac")
```

```
[Out] 2/(cos(x) + 1) + log(cos(x) + 1)
```

Mupad [B]
time = 0.04, size = 14, normalized size = 1.00

$$\ln(\cos(x)+1) + \frac{2}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(cos(x) + 1)^3,x)
```

```
[Out] log(cos(x) + 1) + 2/(cos(x) + 1)
```

3.23 $\int \frac{\sin^3(x)}{(1-\cos(x))^3} \, dx$

Optimal. Leaf size=20

$$-\frac{2}{1-\cos(x)}-\log(1-\cos(x))$$

[Out] -2/(1-cos(x))-ln(1-cos(x))

Rubi [A]

time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2746, 45}

$$-\frac{2}{1-\cos(x)}-\log(1-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(1 - Cos[x])^3,x]
```

```
[Out] -2/(1 - Cos[x]) - Log[1 - Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{(1-\cos(x))^3} \, dx &= \text{Subst}\left(\int \frac{1-x}{(1+x)^2} \, dx, x, -\cos(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right) \, dx, x, -\cos(x)\right)\\
&= -\frac{2}{1-\cos(x)}-\log(1-\cos(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.45

$$-\cot^2\left(\frac{x}{2}\right)-2\log\left(\cos\left(\frac{x}{2}\right)\right)-2\log\left(\tan\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(1 - Cos[x])^3,x]
[Out] -Cot[x/2]^2 - 2*Log[Cos[x/2]] - 2*Log[Tan[x/2]]
```

Maple [A]
time = 0.09, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$\frac{2}{-1+\cos(x)}-\ln\left(-1+\cos\left(x\right)\right)$	17
default	$\frac{2}{-1+\cos(x)}-\ln\left(-1+\cos\left(x\right)\right)$	17
risch	$ix+\frac{4\,e^{ix}}{(e^{ix}-1)^2}-2\ln\left(e^{ix}-1\right)$	32
norman	$\frac{\tan^{11}(\frac{x}{2})+2(\tan^9(\frac{x}{2}))-2(\tan^5(\frac{x}{2}))-(\tan^3(\frac{x}{2}))}{(\tan^2(\frac{x}{2})+1)^3\tan(\frac{x}{2})^5}-2\ln\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan^2\left(\frac{x}{2}\right)+1\right)$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(1-cos(x))^3,x,method=_RETURNVERBOSE)
[Out] 2/(-1+cos(x))-ln(-1+cos(x))
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.80

$$\frac{2}{\cos\left(x\right)-1}-\log\left(\cos\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(1-cos(x))^3,x, algorithm="maxima")
[Out] 2/(cos(x) - 1) - log(cos(x) - 1)
```

Fricas [A]
time = 0.36, size = 22, normalized size = 1.10

$$-\frac{(\cos\left(x\right)-1)\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)-2}{\cos\left(x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(1-cos(x))^3,x, algorithm="fricas")
```

```
[Out] -((cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 2)/(cos(x) - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(14) = 28$.
time = 0.25, size = 126, normalized size = 6.30

$$-\frac{2\log\left(\cos\left(x\right)-1\right)\cos^{2}\left(x\right)}{2\cos^{2}\left(x\right)-4\cos\left(x\right)+2}+\frac{4\log\left(\cos\left(x\right)-1\right)\cos\left(x\right)}{2\cos^{2}\left(x\right)-4\cos\left(x\right)+2}-\frac{2\log\left(\cos\left(x\right)-1\right)}{2\cos^{2}\left(x\right)-4\cos\left(x\right)+2}-\frac{\sin^{2}\left(x\right)}{2\cos^{2}\left(x\right)-4\cos\left(x\right)+2}+\frac{2\cos\left(x\right)}{2\cos^{2}\left(x\right)-4\cos\left(x\right)+2}-\frac{2}{2\cos^{2}\left(x\right)-4\cos\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(1-cos(x))**3,x)
```

```
[Out] -2*log(cos(x) - 1)*cos(x)**2/(2*cos(x)**2 - 4*cos(x) + 2) + 4*log(cos(x) -
1)*cos(x)/(2*cos(x)**2 - 4*cos(x) + 2) - 2*log(cos(x) - 1)/(2*cos(x)**2 - 4
*cos(x) + 2) - sin(x)**2/(2*cos(x)**2 - 4*cos(x) + 2) + 2*cos(x)/(2*cos(x)*
*2 - 4*cos(x) + 2) - 2/(2*cos(x)**2 - 4*cos(x) + 2)
```

Giac [A]
time = 0.43, size = 18, normalized size = 0.90

$$\frac{2}{\cos\left(x\right)-1}-\log\left(-\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(1-cos(x))^3,x, algorithm="giac")
```

```
[Out] 2/(cos(x) - 1) - log(-cos(x) + 1)
```

Mupad [B]
time = 0.04, size = 16, normalized size = 0.80

$$\frac{2}{\cos\left(x\right)-1}-\ln\left(\cos\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(x)^3/(cos(x) - 1)^3,x)
```

```
[Out] 2/(cos(x) - 1) - log(cos(x) - 1)
```

3.24 $\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=104

$$-\frac{a(2a^2-3b^2)\,x}{2b^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4}+\frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2b^3}-\frac{\sin^3(x)}{3b}$$

```
[Out] -1/2*a*(2*a^2-3*b^2)*x/b^4+2*(a-b)^(3/2)*(a+b)^(3/2)*arctan((a-b)^(1/2)*tan
(1/2*x)/(a+b)^(1/2))/b^4+1/2*(2*a^2-2*b^2-a*b*cos(x))*sin(x)/b^3-1/3*sin(x)
^3/b
```

Rubi [A]
time = 0.18, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2774, 2944, 2814, 2738, 211}

$$-\frac{ax(2a^2-3b^2)}{2b^4}+\frac{\sin(x)\,(2(a^2-b^2)-ab\cos(x))}{2b^3}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4}-\frac{\sin^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + b*Cos[x]),x]
```

```
[Out] -1/2*(a*(2*a^2 - 3*b^2)*x)/b^4 + (2*(a - b)^(3/2)*(a + b)^(3/2)*ArcTan[(Sqr
t[a - b]*Tan[x/2])/Sqrt[a + b]])/b^4 + ((2*(a^2 - b^2) - a*b*Cos[x])*Sin[x]
)/(2*b^3) - Sin[x]^3/(3*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
```

```
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4(x)}{a+b\cos(x)}\,dx &= -\frac{\sin^3(x)}{3b}-\frac{\int \frac{(-b-a\cos(x))\sin^2(x)}{a+b\cos(x)}\,dx}{b}\\ &= \frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2b^3}-\frac{\sin^3(x)}{3b}-\frac{\int \frac{b(a^2-2b^2)+a(2a^2-3b^2)\cos(x)}{a+b\cos(x)}\,dx}{2b^3}\\ &= -\frac{a(2a^2-3b^2)\,x}{2b^4}+\frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2b^3}-\frac{\sin^3(x)}{3b}+\frac{(a^2-b^2)^2\int \frac{1}{a+b\cos(x)}\,dx}{b^4}\\ &= -\frac{a(2a^2-3b^2)\,x}{2b^4}+\frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2b^3}-\frac{\sin^3(x)}{3b}+\frac{\left(2(a^2-b^2)^2\right)\operatorname{Subst}\left(\int\right.}{}\\ &= -\frac{a(2a^2-3b^2)\,x}{2b^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4}+\frac{(2(a^2-b^2)-ab\cos}{2b^3}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 96, normalized size = 0.92

$$\frac{-12a^3x+18ab^2x-24(-a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+3b(4a^2-5b^2)\sin(x)-3ab^2\sin(2x)+b^3\sin(3x)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + b*Cos[x]),x]
```

```
[Out] (-12*a^3*x + 18*a*b^2*x - 24*(-a^2 + b^2)^(3/2)*ArcTanh[((a - b)*Tan[x/2])/
Sqrt[-a^2 + b^2]] + 3*b*(4*a^2 - 5*b^2)*Sin[x] - 3*a*b^2*Sin[2*x] + b^3*Sin
[3*x])/(12*b^4)
```

Maple [A]
time = 0.17, size = 152, normalized size = 1.46

method	result
default	$-\frac{2\left(\frac{\left(-a^2b-\frac{1}{2}b^2a+b^3\right)\left(\tan^5\left(\frac{x}{2}\right)\right)+\left(-2a^2b+\frac{10}{3}b^3\right)\left(\tan^3\left(\frac{x}{2}\right)\right)+\left(-a^2b+b^3+\frac{1}{2}b^2a\right)\tan\left(\frac{x}{2}\right)}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}+\frac{a\left(2a^2-3b^2\right)\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{2}\right)}{b^4}+ 2(a-b)^2($ …
risch	$-\frac{a^3x}{b^4}+\frac{3ax}{2b^2}-\frac{ie^{ix}a^2}{2b^3}+\frac{5ie^{ix}}{8b}+\frac{ie^{-ix}a^2}{2b^3}-\frac{5ie^{-ix}}{8b}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{ix}-\frac{i\sqrt{-a^2+b^2}-a}{b}\right)a^2}{b^4}-\sqrt{-a^2+}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^4*(((-a^2*b-1/2*b^2*a+b^3)*tan(1/2*x)^5+(-2*a^2*b+10/3*b^3)*tan(1/2*x)
^3+(-a^2*b+b^3+1/2*b^2*a)*tan(1/2*x))/(tan(1/2*x)^2+1)^3+1/2*a*(2*a^2-3*b^2
)*arctan(tan(1/2*x)))+2*(a-b)^2/b^4*(a+b)^2/((a-b)*(a+b))^(1/2)*arctan((a-b
)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.40, size = 243, normalized size = 2.34

$$\left[-\frac{3\left(a^2-b^2\right)\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(x)+\left(2a^2-b^2\right)\cos(x)^2+2\sqrt{-a^2+b^2}\left(a\cos(x)+b\right)\sin(x)-a^2+2b^2}{b^2\cos(x)^2+2ab\cos(x)+a^2}\right)+3\left(2a^3-3ab^2\right)x-\left(2b^3\cos(x)^2-3ab^2\cos(x)+6a^2b-8b^3\right)\sin(x)}{6b^4},\ \frac{6\left(a^2-b^2\right)^{\frac{3}{2}}\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\sin(x)}\right)-3\left(2a^3-3ab^2\right)x+\left(2b^3\cos(x)^2-3ab^2\cos(x)+6a^2b-8b^3\right)\sin(x)}{6b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2 - b^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos
(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)
^2 + 2*a*b*cos(x) + a^2)) + 3*(2*a^3 - 3*a*b^2)*x - (2*b^3*cos(x)^2 - 3*a*b
^2*cos(x) + 6*a^2*b - 8*b^3)*sin(x))/b^4, 1/6*(6*(a^2 - b^2)^(3/2)*arctan(-
(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))) - 3*(2*a^3 - 3*a*b^2)*x + (2*b^3*c
os(x)^2 - 3*a*b^2*cos(x) + 6*a^2*b - 8*b^3)*sin(x))/b^4]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+b*cos(x)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(86) = 172.
time = 0.51, size = 194, normalized size = 1.87

$$-\frac{(2a^3-3ab^2)x}{2b^4}-\frac{2(a^4-2a^2b^2+b^4)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}+\frac{6a^2\tan\left(\frac{1}{2}x\right)^5+3ab\tan\left(\frac{1}{2}x\right)^5-6b^2\tan\left(\frac{1}{2}x\right)^5+12a^2\tan\left(\frac{1}{2}x\right)^3-20b^2\tan\left(\frac{1}{2}x\right)^3+6a^2\tan\left(\frac{1}{2}x\right)-3ab\tan\left(\frac{1}{2}x\right)-6b^2\tan\left(\frac{1}{2}x\right)}{3\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3 - 3*a*b^2)*x/b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*x/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x))/sqrt(a^2 -
b^2)))/(sqrt(a^2 - b^2)*b^4) + 1/3*(6*a^2*tan(1/2*x)^5 + 3*a*b*tan(1/2*x)^5
 - 6*b^2*tan(1/2*x)^5 + 12*a^2*tan(1/2*x)^3 - 20*b^2*tan(1/2*x)^3 + 6*a^2*t
an(1/2*x) - 3*a*b*tan(1/2*x) - 6*b^2*tan(1/2*x))/((tan(1/2*x)^2 + 1)^3*b^3)
```

Mupad [**B**]
time = 1.11, size = 1677, normalized size = 16.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + b*cos(x)),x)
```

```
[Out] ((4*tan(x/2)^3*(3*a^2 - 5*b^2))/(3*b^3) - (tan(x/2)*(a*b - 2*a^2 + 2*b^2))/
b^3 + (tan(x/2)^5*(a*b + 2*a^2 - 2*b^2))/b^3)/(3*tan(x/2)^2 + 3*tan(x/2)^4
+ tan(x/2)^6 + 1) - (2*atanh((64*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^
2)^(1/2))/(128*a*b^2 + 112*a^2*b - 352*a^3 - 64*b^3 + (16*a^4)/b + (320*a^5
)/b^2 - (112*a^6)/b^3 - (96*a^7)/b^4 + (48*a^8)/b^5) + (144*a^2*tan(x/2)*(b
^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b^4 + 16*a^4*b + 320*a^5 -
```

```
64*b^5 + 112*a^2*b^3 - 352*a^3*b^2 - (112*a^6)/b - (96*a^7)/b^2 + (48*a^8)/
b^3) + (80*a^3*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b
^5 + 320*a^5*b - 112*a^6 - 64*b^6 + 112*a^2*b^4 - 352*a^3*b^3 + 16*a^4*b^2
- (96*a^7)/b + (48*a^8)/b^2) - (144*a^4*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3
*a^4*b^2)^(1/2))/(128*a*b^6 - 112*a^6*b - 96*a^7 - 64*b^7 + 112*a^2*b^5 - 3
52*a^3*b^4 + 16*a^4*b^3 + 320*a^5*b^2 + (48*a^8)/b) + (48*a^5*tan(x/2)*(b^6
 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b^7 - 96*a^7*b + 48*a^8 - 64*
b^8 + 112*a^2*b^6 - 352*a^3*b^5 + 16*a^4*b^4 + 320*a^5*b^3 - 112*a^6*b^2) -
 (192*a*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a*b^3 - 35
2*a^3*b + 16*a^4 - 64*b^4 + 112*a^2*b^2 + (320*a^5)/b - (112*a^6)/b^2 - (96
*a^7)/b^3 + (48*a^8)/b^4))*(-(a + b)^3*(a - b)^3)^(1/2))/b^4 + (a*atan(((a*
(2*a^2 - 3*b^2)*((8*tan(x/2)*(4*a*b^8 - 16*a^8*b + 8*a^9 - 4*b^9 + 7*a^2*b^
7 + 11*a^3*b^6 - 39*a^4*b^5 - 3*a^5*b^4 + 48*a^6*b^3 - 16*a^7*b^2))/b^6 - (
a*((8*(2*a*b^12 - 4*b^13 + 10*a^2*b^11 - 6*a^3*b^10 - 6*a^4*b^9 + 4*a^5*b^8
))/b^9 - (a*tan(x/2)*(2*a^2 - 3*b^2)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*4i
)/b^10)*(2*a^2 - 3*b^2)*1i)/(2*b^4)))/(2*b^4) + (a*(2*a^2 - 3*b^2)*((8*tan(
x/2)*(4*a*b^8 - 16*a^8*b + 8*a^9 - 4*b^9 + 7*a^2*b^7 + 11*a^3*b^6 - 39*a^4*
b^5 - 3*a^5*b^4 + 48*a^6*b^3 - 16*a^7*b^2))/b^6 + (a*((8*(2*a*b^12 - 4*b^13
 + 10*a^2*b^11 - 6*a^3*b^10 - 6*a^4*b^9 + 4*a^5*b^8))/b^9 + (a*tan(x/2)*(2*
a^2 - 3*b^2)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*4i)/b^10)*(2*a^2 - 3*b^2)*
1i)/(2*b^4)))/(2*b^4))/((16*(6*a^10*b - 6*a*b^10 - 4*a^11 + 15*a^2*b^9 + 10
*a^3*b^8 - 49*a^4*b^7 + 8*a^5*b^6 + 59*a^6*b^5 - 26*a^7*b^4 - 31*a^8*b^3 +
18*a^9*b^2))/b^9 + (a*(2*a^2 - 3*b^2)*((8*tan(x/2)*(4*a*b^8 - 16*a^8*b + 8*
a^9 - 4*b^9 + 7*a^2*b^7 + 11*a^3*b^6 - 39*a^4*b^5 - 3*a^5*b^4 + 48*a^6*b^3
- 16*a^7*b^2))/b^6 - (a*((8*(2*a*b^12 - 4*b^13 + 10*a^2*b^11 - 6*a^3*b^10 -
 6*a^4*b^9 + 4*a^5*b^8))/b^9 - (a*tan(x/2)*(2*a^2 - 3*b^2)*(8*a*b^10 - 16*a
^2*b^9 + 8*a^3*b^8)*4i)/b^10)*(2*a^2 - 3*b^2)*1i)/(2*b^4))*1i)/(2*b^4) - (a
*(2*a^2 - 3*b^2)*((8*tan(x/2)*(4*a*b^8 - 16*a^8*b + 8*a^9 - 4*b^9 + 7*a^2*b
^7 + 11*a^3*b^6 - 39*a^4*b^5 - 3*a^5*b^4 + 48*a^6*b^3 - 16*a^7*b^2))/b^6 +
(a*((8*(2*a*b^12 - 4*b^13 + 10*a^2*b^11 - 6*a^3*b^10 - 6*a^4*b^9 + 4*a^5*b^
8))/b^9 + (a*tan(x/2)*(2*a^2 - 3*b^2)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*4
i)/b^10)*(2*a^2 - 3*b^2)*1i)/(2*b^4))*1i)/(2*b^4)))*(2*a^2 - 3*b^2))/b^4
```

3.25 $\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=40

$$-\frac{a\cos(x)}{b^2}+\frac{\cos^2(x)}{2b}+\frac{(a^2-b^2)\log(a+b\cos(x))}{b^3}$$

```
[Out] -a*cos(x)/b^2+1/2*cos(x)^2/b+(a^2-b^2)*ln(a+b*cos(x))/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2747, 711}

$$\frac{(a^2-b^2)\log(a+b\cos(x))}{b^3}-\frac{a\cos(x)}{b^2}+\frac{\cos^2(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + b*Cos[x]),x]
```

```
[Out] -((a*Cos[x])/b^2) + Cos[x]^2/(2*b) + ((a^2 - b^2)*Log[a + b*Cos[x]])/b^3
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(x)}{a+b\cos(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{a+x}\,dx,x,b\cos(x)\right)}{b^3}\\ &= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2+b^2}{a+x}\right)\,dx,x,b\cos(x)\right)}{b^3}\\ &= -\frac{a\cos(x)}{b^2}+\frac{\cos^2(x)}{2b}+\frac{(a^2-b^2)\log(a+b\cos(x))}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 1.00

$$-\frac{a\cos(x)}{b^2}+\frac{\cos(2x)}{4b}+\frac{(a^2-b^2)\log(a+b\cos(x))}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + b*Cos[x]),x]
[Out] -((a*Cos[x])/b^2) + Cos[2*x]/(4*b) + ((a^2 - b^2)*Log[a + b*Cos[x]])/b^3
```

Maple [A]
time = 0.08, size = 39, normalized size = 0.98

method	result	s
default	$-\frac{-\frac{\left(\cos^2(x)\right)b}{2}+a\cos(x)}{b^2}+\frac{(a^2-b^2)\ln(a+b\cos(x))}{b^3}$	3
norman	$\frac{\frac{2a\left(\tan^4\left(\frac{x}{2}\right)\right)}{b^2}-\frac{2a-2b}{3b^2}+\frac{(4a+2b)\left(\tan^6\left(\frac{x}{2}\right)\right)}{3b^2}}{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^3}+\frac{(a-b)(a+b)\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+a+b\right)}{b^3}-\frac{(a-b)(a+b)\ln\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{b^3}$	1
risch	$-\frac{ix\,a^2}{b^3}+\frac{ix}{b}+\frac{e^{2ix}}{8b}-\frac{a\,e^{ix}}{2b^2}-\frac{a\,e^{-ix}}{2b^2}+\frac{e^{-2ix}}{8b}+\frac{\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}+1\right)a^2}{b^3}-\frac{\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}+1\right)}{b}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+b*cos(x)),x,method=_RETURNVERBOSE)
[Out] -1/b^2*(-1/2*cos(x)^2*b+a*cos(x))+(a^2-b^2)*ln(a+b*cos(x))/b^3
```

Maxima [A]
time = 0.26, size = 38, normalized size = 0.95

$$\frac{b\cos(x)^2-2\,a\cos(x)}{2\,b^2}+\frac{(a^2-b^2)\log(b\cos(x)+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*cos(x)),x, algorithm="maxima")
[Out] 1/2*(b*cos(x)^2 - 2*a*cos(x))/b^2 + (a^2 - b^2)*log(b*cos(x) + a)/b^3
```

Fricas [A]
time = 0.39, size = 41, normalized size = 1.02

$$\frac{b^2\cos(x)^2-2\,ab\cos(x)+2\,(a^2-b^2)\log(-b\cos(x)-a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cos(x)^2 - 2*a*b*cos(x) + 2*(a^2 - b^2)*log(-b*cos(x) - a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. $2(34) = 68$.
time = 167.90, size = 1421, normalized size = 35.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*cos(x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x/2) - 1)*tan(x/2)**4/(tan(x/2)**4 + 2*tan(x/2)**2
 + 1) - 2*log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) -
 log(tan(x/2) - 1)/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - log(tan(x/2) + 1)*ta
n(x/2)**4/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - 2*log(tan(x/2) + 1)*tan(x/2)*
*2/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - log(tan(x/2) + 1)/(tan(x/2)**4 + 2*t
an(x/2)**2 + 1) + log(tan(x/2)**2 + 1)*tan(x/2)**4/(tan(x/2)**4 + 2*tan(x/2
)**2 + 1) + 2*log(tan(x/2)**2 + 1)*tan(x/2)**2/(tan(x/2)**4 + 2*tan(x/2)**2
 + 1) + log(tan(x/2)**2 + 1)/(tan(x/2)**4 + 2*tan(x/2)**2 + 1) - 2*tan(x/2)
**2/(tan(x/2)**4 + 2*tan(x/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), ((-sin(x)**2*
cos(x) - 2*cos(x)**3/3)/a, Eq(b, 0)), (-4*tan(x/2)**2/(b*tan(x/2)**4 + 2*b*
tan(x/2)**2 + b) - 2/(b*tan(x/2)**4 + 2*b*tan(x/2)**2 + b), Eq(a, b)), (a**
2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**4/(b**3*tan(x/2)*
*4 + 2*b**3*tan(x/2)**2 + b**3) + 2*a**2*log(-sqrt(-a/(a - b) - b/(a - b))
+ tan(x/2))*tan(x/2)**2/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) + a*
*2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**3*tan(x/2)**4 + 2*b**3
*tan(x/2)**2 + b**3) + a**2*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*ta
n(x/2)**4/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) + 2*a**2*log(sqrt(
-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**2/(b**3*tan(x/2)**4 + 2*b**3*
tan(x/2)**2 + b**3) + a**2*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b*
*3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - a**2*log(tan(x/2)**2 + 1)*tan
(x/2)**4/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - 2*a**2*log(tan(x/
2)**2 + 1)*tan(x/2)**2/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - a**
2*log(tan(x/2)**2 + 1)/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - 2*a
*b*tan(x/2)**2/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - 2*a*b/(b**3
*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - b**2*log(-sqrt(-a/(a - b) - b/(
a - b)) + tan(x/2))*tan(x/2)**4/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b*
*3) - 2*b**2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**2/(b**
3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - b**2*log(-sqrt(-a/(a - b) - b/
(a - b)) + tan(x/2))/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - b**2*
log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**4/(b**3*tan(x/2)**4
+ 2*b**3*tan(x/2)**2 + b**3) - 2*b**2*log(sqrt(-a/(a - b) - b/(a - b)) + ta
n(x/2))*tan(x/2)**2/(b**3*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - b**2*l
og(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**3*tan(x/2)**4 + 2*b**3*tan(
x/2)**2 + b**3) + b**2*log(tan(x/2)**2 + 1)*tan(x/2)**4/(b**3*tan(x/2)**4 +
```

```
 2*b**3*tan(x/2)**2 + b**3) + 2*b**2*log(tan(x/2)**2 + 1)*tan(x/2)**2/(b**3
*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) + b**2*log(tan(x/2)**2 + 1)/(b**3
*tan(x/2)**4 + 2*b**3*tan(x/2)**2 + b**3) - 2*b**2*tan(x/2)**2/(b**3*tan(x/
2)**4 + 2*b**3*tan(x/2)**2 + b**3), True))
```

Giac [A]
time = 0.43, size = 39, normalized size = 0.98

$$\frac{b\cos(x)^2 - 2\,a\cos(x)}{2\,b^2} + \frac{(a^2 - b^2)\log(|b\cos(x) + a|)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] 1/2*(b*cos(x)^2 - 2*a*cos(x))/b^2 + (a^2 - b^2)*log(abs(b*cos(x) + a))/b^3
```

Mupad [B]
time = 0.09, size = 38, normalized size = 0.95

$$\frac{\cos(x)^2}{2\,b} + \frac{\ln(a + b\,\cos(x))\,(a^2 - b^2)}{b^3} - \frac{a\,\cos(x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + b*cos(x)),x)
```

```
[Out] cos(x)^2/(2*b) + (log(a + b*cos(x))*(a^2 - b^2))/b^3 - (a*cos(x))/b^2
```

3.26 $\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=59

$$\frac{ax}{b^2}-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}-\frac{\sin(x)}{b}$$

```
[Out] a*x/b^2-sin(x)/b-2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))*(a-b)^(1/2)*(
a+b)^(1/2)/b^2
```

Rubi [A]
time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2774, 2814, 2738, 211}

$$-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\mathrm{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}+\frac{ax}{b^2}-\frac{\sin(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Cos[x]),x]
```

```
[Out] (a*x)/b^2 - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a
 + b]])/b^2 - Sin[x]/b
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(x)}{a+b\cos(x)}\,dx &= -\frac{\sin(x)}{b}-\frac{\int \frac{-b-a\cos(x)}{a+b\cos(x)}\,dx}{b}\\
&= \frac{ax}{b^2}-\frac{\sin(x)}{b}+\left(1-\frac{a^2}{b^2}\right)\int \frac{1}{a+b\cos(x)}\,dx\\
&= \frac{ax}{b^2}-\frac{\sin(x)}{b}+\left(2\left(1-\frac{a^2}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)\\
&= \frac{ax}{b^2}-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}-\frac{\sin(x)}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 54, normalized size = 0.92

$$
\frac{ax-2\sqrt{-a^2+b^2}\,\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)-b\sin(x)}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Cos[x]),x]
[Out] (a*x - 2*Sqrt[-a^2 + b^2]*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]] - b*
Sin[x])/b^2
```

Maple [A]
time = 0.11, size = 78, normalized size = 1.32

method	result	s
default	$\frac{-\frac{2b\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+2a\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^2}-\frac{2(a+b)(a-b)\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}$	7
risch	$\frac{ax}{b^2}+\frac{ie^{ix}}{2b}-\frac{ie^{-ix}}{2b}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{ix}-\frac{i\sqrt{-a^2+b^2}-a}{b}\right)}{b^2}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{ix}+\frac{i\sqrt{-a^2+b^2}+a}{b}\right)}{b^2}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(-b*tan(1/2*x)/(tan(1/2*x)^2+1)+a*arctan(tan(1/2*x)))-2*(a+b)*(a-b)/b
^2/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.41, size = 154, normalized size = 2.61

$$\left[\frac{2\,ax - 2\,b\sin\left(x\right) + \sqrt{-a^2 + b^2}\,\log\left(\frac{2\,ab\cos\left(x\right) + \left(2\,a^2 - b^2\right)\cos\left(x\right)^2 + 2\,\sqrt{-a^2 + b^2}\,\left(a\cos\left(x\right) + b\right)\sin\left(x\right) - a^2 + 2\,b^2}{b^2\cos\left(x\right)^2 + 2\,ab\cos\left(x\right) + a^2}\right)}{2\,b^2}, \frac{ax - b\sin\left(x\right) - \sqrt{a^2 - b^2}\,\arctan\left(-\frac{a\cos\left(x\right) + b}{\sqrt{a^2 - b^2}\,\sin\left(x\right)}\right)}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x - 2*b*sin(x) + sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b^
2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*
cos(x)^2 + 2*a*b*cos(x) + a^2)))/b^2, (a*x - b*sin(x) - sqrt(a^2 - b^2)*arc
tan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))))/b^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(49) = 98$.
time = 53.72, size = 991, normalized size = 16.80

$$\begin{cases} \tilde{\infty}\left(-\frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right) & \text{for}\: a = 0 \wedge b = 0 \\ \frac{x\tan^2\left(\frac{x}{2}\right)}{b\tan^2\left(\frac{x}{2}\right)+b} + \frac{x}{b\tan^2\left(\frac{x}{2}\right)+b} - \frac{2\tan\left(\frac{x}{2}\right)}{b\tan^2\left(\frac{x}{2}\right)+b} & \text{for}\: a = b \\ -\frac{x\tan^2\left(\frac{x}{2}\right)}{b\tan^2\left(\frac{x}{2}\right)+b} - \frac{x}{b\tan^2\left(\frac{x}{2}\right)+b} - \frac{2\tan\left(\frac{x}{2}\right)}{b\tan^2\left(\frac{x}{2}\right)+b} & \text{for}\: a = -b \\ \frac{\frac{x\sin^2\left(x\right)}{2} + \frac{x\cos^2\left(x\right)}{2} - \frac{\sin\left(x\right)\cos\left(x\right)}{2}}{a} & \text{for}\: b = 0 \\ \frac{ax\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}\tan^2\left(\frac{x}{2}\right)}{b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} + \frac{ax\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{a\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)\tan^2\left(\frac{x}{2}\right)}{b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{a\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)}{b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} + \frac{a\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)\tan^2\left(\frac{x}{2}\right)}{b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} + \frac{a\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)}{b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{2b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}\tan\left(\frac{x}{2}\right)}{b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{b\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)\tan^2\left(\frac{x}{2}\right)}{b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{b\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)}{b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} + \frac{b\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)\tan^2\left(\frac{x}{2}\right)}{b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} + \frac{b\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)}{b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*cos(x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**2 + 1) - log(tan(
x/2) - 1)/(tan(x/2)**2 + 1) + log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)**2 +
1) + log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - 2*tan(x/2)/(tan(x/2)**2 + 1)), E
q(a, 0) & Eq(b, 0)), (x*tan(x/2)**2/(b*tan(x/2)**2 + b) + x/(b*tan(x/2)**2
+ b) - 2*tan(x/2)/(b*tan(x/2)**2 + b), Eq(a, b)), (-x*tan(x/2)**2/(b*tan(x/
2)**2 + b) - x/(b*tan(x/2)**2 + b) - 2*tan(x/2)/(b*tan(x/2)**2 + b), Eq(a,
-b)), ((x*sin(x)**2/2 + x*cos(x)**2/2 - sin(x)*cos(x)/2)/a, Eq(b, 0)), (a*x
*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2/(b**2*sqrt(-a/(a - b) - b/(a - b)
)*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))) + a*x*sqrt(-a/(a - b) -
b/(a - b))/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a
 - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x
/2)**2/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b
) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**2*sqr
t(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b)))
+ a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**2/(b**2*sqrt(-a/
(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))) + a*l
og(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**2*sqrt(-a/(a - b) - b/(a -
b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*b*sqrt(-a/(a - b)
- b/(a - b))*tan(x/2)/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2
*sqrt(-a/(a - b) - b/(a - b))) - b*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(
x/2))*tan(x/2)**2/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqr
t(-a/(a - b) - b/(a - b))) - b*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2)
)/(b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b
/(a - b))) + b*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**2/(b*
*2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a -
 b))) + b*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(b**2*sqrt(-a/(a - b
) - b/(a - b))*tan(x/2)**2 + b**2*sqrt(-a/(a - b) - b/(a - b))), True))
```

Giac [A]
time = 0.45, size = 90, normalized size = 1.53

$$\frac{ax}{b^2} + \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2} - \frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] a*x/b^2 + 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
x) - b*tan(1/2*x))/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2 - 2*tan(1/2*x)/((t
an(1/2*x)^2 + 1)*b)
```

Mupad [B]

time = 0.49, size = 74, normalized size = 1.25

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)\sqrt{b^2-a^2}}{a\,\cos\left(\frac{x}{2}\right)+b\,\cos\left(\frac{x}{2}\right)}\right)\sqrt{b^2-a^2}}{b^2}-\frac{\sin\left(x\right)}{b}+\frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b*cos(x)),x)
[Out] (2*atanh((sin(x/2)*(b^2 - a^2)^(1/2))/(a*cos(x/2) + b*cos(x/2)))*(b^2 - a^2
)^(1/2))/b^2 - sin(x)/b + (2*a*atan(sin(x/2)/cos(x/2)))/b^2
```

3.27 $\int \frac{\sin(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\cos(x))}{b}$$

[Out] -ln(a+b*cos(x))/b

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2747, 31}

$$-\frac{\log(a+b\cos(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(a + b*Cos[x]),x]

[Out] -(Log[a + b*Cos[x]]/b)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a+b\cos(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cos(x)\right)}{b}\\ &= -\frac{\log(a+b\cos(x))}{b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 12, normalized size = 1.00

$$-\frac{\log(a+b\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Cos[x]),x]
[Out] -(Log[a + b*Cos[x]]/b)
```

Maple [A]
time = 0.04, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+b\cos(x))}{b}$	13
default	$-\frac{\ln(a+b\cos(x))}{b}$	13
risch	$\frac{ix}{b}-\frac{\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}+1\right)}{b}$	33
norman	$\frac{\ln(\tan^2(\frac{x}{2})+1)}{b}-\frac{\ln(a(\tan^2(\frac{x}{2}))-b(\tan^2(\frac{x}{2}))+a+b)}{b}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*cos(x)),x,method=_RETURNVERBOSE)
[Out] -ln(a+b*cos(x))/b
```

Maxima [A]
time = 0.27, size = 12, normalized size = 1.00

$$-\frac{\log\left(b\cos\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="maxima")
[Out] -log(b*cos(x) + a)/b
```

Fricas [A]
time = 0.37, size = 15, normalized size = 1.25

$$-\frac{\log\left(-b\cos\left(x\right)-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="fricas")
[Out] -log(-b*cos(x) - a)/b
```

Sympy [A]
time = 0.16, size = 17, normalized size = 1.42

$$\begin{cases} -\frac{\log\left(\frac{a}{b}+\cos\left(x\right)\right)}{b} & \text{for}\ b\neq 0 \\ -\frac{\cos\left(x\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x)
```

```
[Out] Piecewise((-log(a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

Giac [A]
time = 0.50, size = 13, normalized size = 1.08

$$-\frac{\log\left(|b\cos\left(x\right)+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -log(abs(b*cos(x) + a))/b
```

Mupad [B]
time = 0.04, size = 12, normalized size = 1.00

$$-\frac{\ln\left(a+b\,\cos\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*cos(x)),x)
```

```
[Out] -log(a + b*cos(x))/b
```

3.28 $\int \frac{1}{a+b\cos(x)}\,dx$

Optimal. Leaf size=42

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] 2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2738, 211}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x])^(-1),x]
```

```
[Out] (2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cos(x)}\,dx &= 2\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 0.98

$$-\frac{2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x])^(-1),x]
[Out] (-2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
```

Maple [A]
time = 0.06, size = 36, normalized size = 0.86

method	result	size
default	$\frac{2\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}$	36
risch	$-\frac{\ln\left(e^{ix}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{\ln\left(e^{ix}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)),x,method=_RETURNVERBOSE)
[Out] 2/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.38, size = 137, normalized size = 3.26

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+(2\,a^2-b^2)\cos(x)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)}{2\,(a^2-b^2)},\ \frac{\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\sin(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos(x)^2 + 2*sqrt(
-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*cos(
x) + a^2))/(a^2 - b^2), arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x)))/sq
rt(a^2 - b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(34) = 68$.
time = 1.79, size = 144, normalized size = 3.43

$$\begin{cases} \tilde{\infty}\left(-\log\left(\tan\left(\frac{x}{2}\right)-1\right)+\log\left(\tan\left(\frac{x}{2}\right)+1\right)\right) & \text{for } a=0 \wedge b=0 \\ \frac{1}{b\tan\left(\frac{x}{2}\right)} & \text{for } a=-b \\ \frac{\tan\left(\frac{x}{2}\right)}{b} & \text{for } a=b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x/2) - 1) + log(tan(x/2) + 1)), Eq(a, 0) & Eq(b, 0
)), (1/(b*tan(x/2)), Eq(a, -b)), (tan(x/2)/b, Eq(a, b)), (log(-sqrt(-a/(a -
 b) - b/(a - b)) + tan(x/2))/(a*sqrt(-a/(a - b) - b/(a - b)) - b*sqrt(-a/(a
 - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(a*sqrt(
-a/(a - b) - b/(a - b)) - b*sqrt(-a/(a - b) - b/(a - b))), True))
```

Giac [A]
time = 0.44, size = 61, normalized size = 1.45

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x))/sqrt(a^2 - b^2)))/sqrt(a^2 - b^2)
```

Mupad [B]
time = 0.48, size = 38, normalized size = 0.90

$$\frac{2\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)(2a-2b)}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)),x)
[Out] (2*atan((tan(x/2)*(2*a - 2*b))/(2*(a^2 - b^2)^(1/2))))/(a^2 - b^2)^(1/2)
```

3.29 $\int \frac{\csc(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=53

$$\frac{\log(1-\cos(x))}{2(a+b)}-\frac{\log(1+\cos(x))}{2(a-b)}+\frac{b\log(a+b\cos(x))}{a^2-b^2}$$

```
[Out] 1/2*ln(1-cos(x))/(a+b)-1/2*ln(cos(x)+1)/(a-b)+b*ln(a+b*cos(x))/(a^2-b^2)
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2747, 720, 31, 647}

$$\frac{b\log(a+b\cos(x))}{a^2-b^2}+\frac{\log(1-\cos(x))}{2(a+b)}-\frac{\log(\cos(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Cos[x]),x]
```

```
[Out] Log[1 - Cos[x]]/(2*(a + b)) - Log[1 + Cos[x]]/(2*(a - b)) + (b*Log[a + b*Co
s[x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{a+b\cos(x)}\,dx &= -\left(b\text{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cos(x)\right)\right)\\ &= \frac{b\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cos(x)\right)}{a^2-b^2}+\frac{b\text{Subst}\left(\int \frac{-a+x}{b^2-x^2}\,dx, x, b\cos(x)\right)}{a^2-b^2}\\ &= \frac{b\log(a+b\cos(x))}{a^2-b^2}+\frac{\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\cos(x)\right)}{2(a-b)}-\frac{\text{Subst}\left(\int \frac{1}{b-x}\,dx, x, b\cos(x)\right)}{2(a+b)}\\ &= \frac{\log(1-\cos(x))}{2(a+b)}-\frac{\log(1+\cos(x))}{2(a-b)}+\frac{b\log(a+b\cos(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 0.94

$$\frac{(a-b)\log(1-\cos(x))-(a+b)\log(1+\cos(x))+2b\log(a+b\cos(x))}{2(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Cos[x]),x]
```

```
[Out] ((a - b)*Log[1 - Cos[x]] - (a + b)*Log[1 + Cos[x]] + 2*b*Log[a + b*Cos[x]])
/(2*(a - b)*(a + b))
```

Maple [A]
time = 0.10, size = 54, normalized size = 1.02

method	result	size
norman	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a+b}+\frac{b\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+a+b\right)}{a^2-b^2}$	47
default	$\frac{b\ln(a+b\cos(x))}{(a-b)(a+b)}+\frac{\ln(-1+\cos(x))}{2a+2b}-\frac{\ln(\cos(x)+1)}{2a-2b}$	54
risch	$\frac{ix}{a-b}-\frac{ix}{a+b}-\frac{2ixb}{a^2-b^2}-\frac{\ln\left(e^{ix}+1\right)}{a-b}+\frac{\ln\left(e^{ix}-1\right)}{a+b}+\frac{b\ln\left(e^{2ix}+\frac{2a\,e^{ix}}{b}+1\right)}{a^2-b^2}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] b/(a-b)/(a+b)*ln(a+b*cos(x))+1/(2*a+2*b)*ln(-1+cos(x))-1/(2*a-2*b)*ln(cos(x
)+1)
```

Maxima [A]
time = 0.29, size = 47, normalized size = 0.89

$$\frac{b\log\left(b\cos\left(x\right)+a\right)}{a^{2}-b^{2}}-\frac{\log\left(\cos\left(x\right)+1\right)}{2\left(a-b\right)}+\frac{\log\left(\cos\left(x\right)-1\right)}{2\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] b*log(b*cos(x) + a)/(a^2 - b^2) - 1/2*log(cos(x) + 1)/(a - b) + 1/2*log(cos
(x) - 1)/(a + b)
```

Fricas [A]
time = 0.42, size = 52, normalized size = 0.98

$$\frac{2\,b\log\left(-b\cos\left(x\right)-a\right)-\left(a+b\right)\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+\left(a-b\right)\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)}{2\left(a^{2}-b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*log(-b*cos(x) - a) - (a + b)*log(1/2*cos(x) + 1/2) + (a - b)*log(-
1/2*cos(x) + 1/2))/(a^2 - b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(x\right)}{a+b\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cos(x)),x)
```

```
[Out] Integral(csc(x)/(a + b*cos(x)), x)
```

Giac [A]
time = 0.42, size = 54, normalized size = 1.02

$$\frac{b^{2}\log\left(\left|b\cos\left(x\right)+a\right|\right)}{a^{2}b-b^{3}}-\frac{\log\left(\cos\left(x\right)+1\right)}{2\left(a-b\right)}+\frac{\log\left(-\cos\left(x\right)+1\right)}{2\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(b*cos(x) + a))/(a^2*b - b^3) - 1/2*log(cos(x) + 1)/(a - b) + 1/
2*log(-cos(x) + 1)/(a + b)
```

Mupad [B]
time = 0.21, size = 52, normalized size = 0.98

$$\frac{\ln\left(\cos\left(x\right)-1\right)}{2\,\left(a+b\right)}-\frac{\ln\left(\cos\left(x\right)+1\right)}{2\,\left(a-b\right)}+\frac{b\,\ln\left(a+b\,\cos\left(x\right)\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + b*cos(x))),x)
```

```
[Out] log(cos(x) - 1)/(2*(a + b)) - log(cos(x) + 1)/(2*(a - b)) + (b*log(a + b*co
s(x)))/(a^2 - b^2)
```

3.30 $\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=67

$$-\frac{2b^2\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cos(x))\csc(x)}{a^2-b^2}$$

```
[Out] -2*b^2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)+(
b-a*cos(x))*csc(x)/(a^2-b^2)
```

Rubi [A]
time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2775, 12, 2738, 211}

$$\frac{\csc(x)(b-a\cos(x))}{a^2-b^2}-\frac{2b^2\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Cos[x]),x]
```

```
[Out] (-2*b^2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)^
(3/2)) + ((b - a*Cos[x])*Csc[x])/(a^2 - b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
```

```
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{a+b\cos(x)}\,dx &= \frac{(b-a\cos(x))\csc(x)}{a^2-b^2}+\frac{\int \frac{b^2}{a+b\cos(x)}\,dx}{-a^2+b^2}\\ &= \frac{(b-a\cos(x))\csc(x)}{a^2-b^2}-\frac{b^2\int \frac{1}{a+b\cos(x)}\,dx}{a^2-b^2}\\ &= \frac{(b-a\cos(x))\csc(x)}{a^2-b^2}-\frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2}\\ &= -\frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cos(x))\csc(x)}{a^2-b^2}\end{aligned}$$

Mathematica [A]
time = 0.35, size = 66, normalized size = 0.99

$$-\frac{2b^2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}+\frac{(b-a\cos(x))\csc(x)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Cos[x]),x]
```

```
[Out] (-2*b^2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) +
((b - a*Cos[x])*Csc[x])/(a^2 - b^2)
```

Maple [A]
time = 0.13, size = 78, normalized size = 1.16

method	result	size
default	$\frac{\tan\left(\frac{x}{2}\right)}{2a-2b}-\frac{1}{2(a+b)\tan\left(\frac{x}{2}\right)}-\frac{2b^2\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}$	78

risch	$-\frac{2i(-b\,e^{ix}+a)}{(e^{2ix}-1)(a^2-b^2)}+\frac{b^2\ln\left(e^{ix}+\frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}-\frac{b^2\ln\left(e^{ix}+\frac{-ia^2+ib^2+a\sqrt{-a^2+b^2}}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}$	186

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a-b)*tan(1/2*x)-1/2/(a+b)/tan(1/2*x)-2/(a-b)/(a+b)*b^2/((a-b)*(a+b))^(
1/2)*arctan((a-b)*tan(1/2*x)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.40, size = 230, normalized size = 3.43

$$\left[\frac{\sqrt{-a^2+b^2}\,b^2\log\left(\frac{2\,ab\cos(x)+(2\,a^2-b^2)\cos(x)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)\sin(x)+2\,a^2b-2\,b^3-2\,(a^3-ab^2)\cos(x)}{2\,(a^4-2\,a^2b^2+b^4)\sin(x)},-\frac{\sqrt{a^2-b^2}\,b^2\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\,\sin(x)}\right)\sin(x)-a^2b+b^3+(a^3-ab^2)\cos(x)}{(a^4-2\,a^2b^2+b^4)\sin(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*b^2*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos(x)^2 + 2*s
qrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*
cos(x) + a^2))*sin(x) + 2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(x))/((a^4 - 2
*a^2*b^2 + b^4)*sin(x)), -(sqrt(a^2 - b^2)*b^2*arctan(-(a*cos(x) + b)/(sqrt
(a^2 - b^2)*sin(x)))*sin(x) - a^2*b + b^3 + (a^3 - a*b^2)*cos(x))/((a^4 - 2
*a^2*b^2 + b^4)*sin(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2(x)}{a+b\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*cos(x)),x)
```

```
[Out] Integral(csc(x)**2/(a + b*cos(x)), x)
```

Giac [A]
time = 0.55, size = 91, normalized size = 1.36

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)b^2}{(a^2-b^2)^{\frac{3}{2}}}+\frac{\tan\left(\frac{1}{2}x\right)}{2(a-b)}-\frac{1}{2(a+b)\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan
(1/2*x))/sqrt(a^2 - b^2)))*b^2/(a^2 - b^2)^(3/2) + 1/2*tan(1/2*x)/(a - b) -
 1/2/((a + b)*tan(1/2*x))
```

Mupad [B]
time = 0.47, size = 86, normalized size = 1.28

$$\frac{\tan\left(\frac{x}{2}\right)}{2a-2b}-\frac{2b^2\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)\left(a^2-b^2\right)}{(a+b)^{3/2}\sqrt{a-b}}\right)}{(a+b)^{3/2}(a-b)^{3/2}}-\frac{a-b}{\tan\left(\frac{x}{2}\right)(a+b)(2a-2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + b*cos(x))),x)
```

```
[Out] tan(x/2)/(2*a - 2*b) - (2*b^2*atan((tan(x/2)*(a^2 - b^2))/((a + b)^(3/2)*(a
 - b)^(1/2))))/((a + b)^(3/2)*(a - b)^(3/2)) - (a - b)/(tan(x/2)*(a + b)*(2
*a - 2*b))
```

3.31 $\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=92

$$\frac{(b-a\cos(x))\csc^2(x)}{2(a^2-b^2)}+\frac{(a+2b)\log(1-\cos(x))}{4(a+b)^2}-\frac{(a-2b)\log(1+\cos(x))}{4(a-b)^2}-\frac{b^3\log(a+b\cos(x))}{(a^2-b^2)^2}$$

```
[Out] 1/2*(b-a*cos(x))*csc(x)^2/(a^2-b^2)+1/4*(a+2*b)*ln(1-cos(x))/(a+b)^2-1/4*(a
-2*b)*ln(cos(x)+1)/(a-b)^2-b^3*ln(a+b*cos(x))/(a^2-b^2)^2
```

Rubi [A]
time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2747, 755, 815}

$$\frac{\csc^2(x)(b-a\cos(x))}{2(a^2-b^2)}-\frac{b^3\log(a+b\cos(x))}{(a^2-b^2)^2}+\frac{(a+2b)\log(1-\cos(x))}{4(a+b)^2}-\frac{(a-2b)\log(\cos(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + b*Cos[x]),x]
```

```
[Out] ((b - a*Cos[x])*Csc[x]^2)/(2*(a^2 - b^2)) + ((a + 2*b)*Log[1 - Cos[x]])/(4*
(a + b)^2) - ((a - 2*b)*Log[1 + Cos[x]])/(4*(a - b)^2) - (b^3*Log[a + b*Cos
[x]])/(a^2 - b^2)^2
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(x)}{a+b\cos(x)}\,dx = -\left(b^3\text{Subst}\left(\int \frac{1}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\cos(x)\right)\right)$$

$$= \frac{(b-a\cos(x))\csc^2(x)}{2\left(a^2-b^2\right)} - \frac{b\text{Subst}\left(\int \frac{a^2-2b^2+ax}{(a+x)(b^2-x^2)}\,dx, x, b\cos(x)\right)}{2\left(a^2-b^2\right)}$$

$$= \frac{(b-a\cos(x))\csc^2(x)}{2\left(a^2-b^2\right)} - \frac{b\text{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right)\,dx, x, b \ldots}{2\left(a^2-b^2\right)}$$

$$= \frac{(b-a\cos(x))\csc^2(x)}{2\left(a^2-b^2\right)} + \frac{(a+2b)\log(1-\cos(x))}{4(a+b)^2} - \frac{(a-2b)\log(1+\cos(x))}{4(a-b)^2} - \frac{b^3\log(\ldots}{(a \ldots}$$

Mathematica [A]
time = 0.46, size = 99, normalized size = 1.08

$$\frac{1}{8}\left(-\frac{\csc^2\left(\frac{x}{2}\right)}{a+b} - \frac{4(a-2b)\log\left(\cos\left(\frac{x}{2}\right)\right)}{(a-b)^2} - \frac{8b^3\log(a+b\cos(x))}{\left(a^2-b^2\right)^2} + \frac{4(a+2b)\log\left(\sin\left(\frac{x}{2}\right)\right)}{(a+b)^2} + \frac{\sec^2\left(\frac{x}{2}\right)}{a-b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + b*Cos[x]),x]
```

```
[Out] (-(Csc[x/2]^2/(a + b)) - (4*(a - 2*b)*Log[Cos[x/2]])/(a - b)^2 - (8*b^3*Log
[a + b*Cos[x]])/(a^2 - b^2)^2 + (4*(a + 2*b)*Log[Sin[x/2]])/(a + b)^2 + Sec
[x/2]^2/(a - b))/8
```

Maple [A]
time = 0.16, size = 96, normalized size = 1.04

method	result
default	$-\frac{b^3\ln(a+b\cos(x))}{(a+b)^2(a-b)^2} + \frac{1}{(4a+4b)(-1+\cos(x))} + \frac{(a+2b)\ln(-1+\cos(x))}{4(a+b)^2} + \frac{1}{(4a-4b)(\cos(x)+1)} + \frac{(-a+2b)\ln(\cos(x)+1)}{4(a-b)^2}$
norman	$\frac{-\frac{1}{8(a+b)} + \frac{\tan^4\left(\frac{x}{2}\right)}{8a-8b}}{\tan\left(\frac{x}{2}\right)^2} - \frac{b^3\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right) - b\left(\tan^2\left(\frac{x}{2}\right)\right) + a + b\right)}{a^4-2a^2b^2+b^4} + \frac{(a+2b)\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a^2+4ab+2b^2}$
risch	$-\frac{ixa}{2(a^2+2ab+b^2)} - \frac{ixb}{a^2+2ab+b^2} + \frac{ixa}{2a^2-4ab+2b^2} - \frac{ixb}{a^2-2ab+b^2} + \frac{2ix\,b^3}{a^4-2a^2b^2+b^4} - \frac{a\,e^{3ix} - 2b\,e^{2ix} + a\,e^{ix}}{\left(e^{2ix}-1\right)^2\left(-a^2+b^2\right)} + \frac{\ln\left(e^{ix}-1\right)a}{2a^2+4ab+2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/(a+b)^2/(a-b)^2*ln(a+b*cos(x))+1/(4*a+4*b)/(-1+cos(x))+1/4*(a+2*b)/(a+
b)^2*ln(-1+cos(x))+1/(4*a-4*b)/(cos(x)+1)+1/4/(a-b)^2*(-a+2*b)*ln(cos(x)+1)
```

Maxima [A]
time = 0.37, size = 115, normalized size = 1.25

$$-\frac{b^3\log\left(b\cos\left(x\right)+a\right)}{a^4-2\,a^2b^2+b^4}-\frac{(a-2\,b)\log\left(\cos\left(x\right)+1\right)}{4\,(a^2-2\,ab+b^2)}+\frac{(a+2\,b)\log\left(\cos\left(x\right)-1\right)}{4\,(a^2+2\,ab+b^2)}+\frac{a\cos\left(x\right)-b}{2\,\left((a^2-b^2)\cos\left(x\right)^2-a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log(b*cos(x) + a)/(a^4 - 2*a^2*b^2 + b^4) - 1/4*(a - 2*b)*log(cos(x) +
 1)/(a^2 - 2*a*b + b^2) + 1/4*(a + 2*b)*log(cos(x) - 1)/(a^2 + 2*a*b + b^2)
 + 1/2*(a*cos(x) - b)/((a^2 - b^2)*cos(x)^2 - a^2 + b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(87) = 174.
time = 0.42, size = 181, normalized size = 1.97

$$\frac{2\,a^2b-2\,b^3-2\,(a^3-ab^2)\cos\left(x\right)+4\,\left(b^3\cos\left(x\right)^2-b^3\right)\log\left(-b\cos\left(x\right)-a\right)-\left(a^3-3\,ab^2-2\,b^3-(a^3-3\,ab^2-2\,b^3)\cos\left(x\right)^2\right)\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\left(a^3-3\,ab^2+2\,b^3-(a^3-3\,ab^2+2\,b^3)\cos\left(x\right)^2\right)\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)}{4\,\left(a^4-2\,a^2b^2+b^4-(a^4-2\,a^2b^2+b^4)\cos\left(x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(x) + 4*(b^3*cos(x)^2 - b^3)*log(
-b*cos(x) - a) - (a^3 - 3*a*b^2 - 2*b^3 - (a^3 - 3*a*b^2 - 2*b^3)*cos(x)^2)
*log(1/2*cos(x) + 1/2) + (a^3 - 3*a*b^2 + 2*b^3 - (a^3 - 3*a*b^2 + 2*b^3)*c
os(x)^2)*log(-1/2*cos(x) + 1/2))/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2
+ b^4)*cos(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^3\left(x\right)}{a+b\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*cos(x)),x)
```

```
[Out] Integral(csc(x)**3/(a + b*cos(x)), x)
```

Giac [A]
time = 0.47, size = 136, normalized size = 1.48

$$-\frac{b^4\log\left(|b\cos\left(x\right)+a|\right)}{a^4b-2\,a^2b^3+b^5}-\frac{(a-2\,b)\log\left(\cos\left(x\right)+1\right)}{4\,(a^2-2\,ab+b^2)}+\frac{(a+2\,b)\log\left(-\cos\left(x\right)+1\right)}{4\,(a^2+2\,ab+b^2)}-\frac{a^2b-b^3-(a^3-ab^2)\cos\left(x\right)}{2\,(a+b)^2(a-b)^2(\cos\left(x\right)+1)(\cos\left(x\right)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -b^4*log(abs(b*cos(x) + a))/(a^4*b - 2*a^2*b^3 + b^5) - 1/4*(a - 2*b)*log(c
os(x) + 1)/(a^2 - 2*a*b + b^2) + 1/4*(a + 2*b)*log(-cos(x) + 1)/(a^2 + 2*a*
b + b^2) - 1/2*(a^2*b - b^3 - (a^3 - a*b^2)*cos(x))/((a + b)^2*(a - b)^2*(c
os(x) + 1)*(cos(x) - 1))
```

Mupad [B]
time = 0.51, size = 112, normalized size = 1.22

$$\ln\left(\cos\left(x\right)-1\right)\,\left(\frac{b}{4\,{\left(a+b\right)}^2}+\frac{1}{4\,\left(a+b\right)}\right)+\frac{\frac{b}{2\,\left(a^2-b^2\right)}-\frac{a\,\cos\left(x\right)}{2\,\left(a^2-b^2\right)}}{{\sin\left(x\right)}^2}-\frac{b^3\,\ln\left(a+b\,\cos\left(x\right)\right)}{a^4-2\,a^2\,b^2+b^4}-\frac{\ln\left(\cos\left(x\right)+1\right)\,\left(a-2\,b\right)}{4\,{\left(a-b\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + b*cos(x))),x)
```

```
[Out] log(cos(x) - 1)*(b/(4*(a + b)^2) + 1/(4*(a + b))) + (b/(2*(a^2 - b^2)) - (a
*cos(x))/(2*(a^2 - b^2)))/sin(x)^2 - (b^3*log(a + b*cos(x)))/(a^4 + b^4 - 2
*a^2*b^2) - (log(cos(x) + 1)*(a - 2*b))/(4*(a - b)^2)
```

3.32 $\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=110

$$\frac{2b^4\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{(3b^3+a(2a^2-5b^2)\cos(x))\csc(x)}{3\left(a^2-b^2\right)^2}+\frac{(b-a\cos(x))\csc^3(x)}{3\left(a^2-b^2\right)}$$

```
[Out] 2*b^4*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)-1/
3*(3*b^3+a*(2*a^2-5*b^2)*cos(x))*csc(x)/(a^2-b^2)^2+1/3*(b-a*cos(x))*csc(x)
^3/(a^2-b^2)
```

Rubi [A]
time = 0.18, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2775, 2945, 12, 2738, 211}

$$\frac{\csc^3(x)(b-a\cos(x))}{3\left(a^2-b^2\right)}-\frac{\csc(x)\left(a(2a^2-5b^2)\cos(x)+3b^3\right)}{3\left(a^2-b^2\right)^2}+\frac{2b^4\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + b*Cos[x]),x]
```

```
[Out] (2*b^4*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(
5/2)) - ((3*b^3 + a*(2*a^2 - 5*b^2)*Cos[x])*Csc[x])/(3*(a^2 - b^2)^2) + ((b
 - a*Cos[x])*Csc[x]^3)/(3*(a^2 - b^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(x)}{a+b\cos(x)}\,dx &= \frac{(b-a\cos(x))\csc^3(x)}{3(a^2-b^2)} - \frac{\int \frac{(-2a^2+3b^2-2ab\cos(x))\csc^2(x)}{a+b\cos(x)}\,dx}{3(a^2-b^2)} \\
&= -\frac{(3b^3+a(2a^2-5b^2)\cos(x))\csc(x)}{3(a^2-b^2)^2} + \frac{(b-a\cos(x))\csc^3(x)}{3(a^2-b^2)} + \frac{\int \frac{3b^4}{a+b\cos(x)}\,dx}{3(a^2-b^2)^2} \\
&= -\frac{(3b^3+a(2a^2-5b^2)\cos(x))\csc(x)}{3(a^2-b^2)^2} + \frac{(b-a\cos(x))\csc^3(x)}{3(a^2-b^2)} + \frac{b^4\int \frac{1}{a+b\cos(x)}\,dx}{(a^2-b^2)^2} \\
&= -\frac{(3b^3+a(2a^2-5b^2)\cos(x))\csc(x)}{3(a^2-b^2)^2} + \frac{(b-a\cos(x))\csc^3(x)}{3(a^2-b^2)} + \frac{(2b^4)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-}\right.}{(a^2-b} \\
&= \frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{(3b^3+a(2a^2-5b^2)\cos(x))\csc(x)}{3(a^2-b^2)^2} + \frac{(b-a\cos(x))\csc^3}{3(a^2-b^2)}
\end{aligned}$$

Mathematica [A]
time = 0.70, size = 112, normalized size = 1.02

$$-\frac{2b^4\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{((-6a^3+9ab^2)\cos(x)+6b^3\cos(2x)+(2a^2-5b^2)(2b+a\cos(3x)))\csc^3(x)}{12(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + b*Cos[x]),x]
```

```
[Out] (-2*b^4*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) +
(((-6*a^3 + 9*a*b^2)*Cos[x] + 6*b^3*Cos[2*x] + (2*a^2 - 5*b^2)*(2*b + a*Cos
[3*x]))*Csc[x]^3)/(12*(a - b)^2*(a + b)^2)
```

Maple [A]
time = 0.21, size = 127, normalized size = 1.15

method	result
default	$\frac{\frac{a\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3}-\frac{b\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3}+3a\tan\left(\frac{x}{2}\right)-5b\tan\left(\frac{x}{2}\right)}{8(a-b)^{2}}+\frac{2b^{4}\arctan\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^{2}(a+b)^{2}\sqrt{(a-b)(a+b)}}-\frac{1}{24(a+b)\tan\left(\frac{x}{2}\right)^{3}}-\frac{3a+5b}{8(a+b)^{2}\tan(}$
risch	$-\frac{2i\left(3b^{3}e^{5ix}-3a\,b^{2}e^{4ix}+4a^{2}b\,e^{3ix}-10b^{3}e^{3ix}-6a^{3}e^{2ix}+12a\,b^{2}e^{2ix}+3b^{3}e^{ix}+2a^{3}-5b^{2}a\right)}{3\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\left(e^{2ix}-1\right)^{3}}-\frac{b^{4}\ln\left(e^{ix}+\frac{ia^{2}-ib^{2}+a\sqrt{-a^{2}+b^{2}}}{b\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}\,(a+b)^{2}(a-b)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/(a-b)^2*(1/3*a*tan(1/2*x)^3-1/3*b*tan(1/2*x)^3+3*a*tan(1/2*x)-5*b*tan(1
/2*x))+2/(a-b)^2/(a+b)^2*b^4/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*x)/((
a-b)*(a+b))^(1/2))-1/24/(a+b)/tan(1/2*x)^3-1/8*(3*a+5*b)/(a+b)^2/tan(1/2*x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(97) = 194$.
time = 0.39, size = 459, normalized size = 4.17

$$\left[\frac{2a^{4}b-10a^{2}b^{3}+8b^{5}+2\left(2a^{5}-7a^{3}b^{2}+5ab^{4}\right)\cos\left(x\right)^{3}+3\left(b^{4}\cos\left(x\right)^{2}-b^{4}\right)\sqrt{-a^{2}+b^{2}}\log\left(\frac{2ab\cos(x)+(2a^{2}-b^{2})\cos(x)^{2}+2\sqrt{-a^{2}+b^{2}}(a\cos(x)+b)\sin(x)-a^{2}+2b^{2}}{b^{2}\cos(x)^{2}+2ab\cos(x)+a^{2}}\right)\sin\left(x\right)+6\left(a^{2}b^{3}-b^{5}\right)\cos\left(x\right)^{2}-6\left(a^{5}-3a^{3}b^{2}+2ab^{4}\right)\cos\left(x\right)}{6\left(a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}-\left(a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}\right)\cos\left(x\right)^{2}\right)\sin\left(x\right)},\frac{a^{4}b-5a^{2}b^{3}+4b^{5}+\left(2a^{5}-7a^{3}b^{2}+5ab^{4}\right)\cos\left(x\right)^{3}-3\left(b^{4}\cos\left(x\right)^{2}-b^{4}\right)\sqrt{a^{2}-b^{2}}\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^{2}-b^{2}}\sin(x)}\right)\sin\left(x\right)+3\left(a^{2}b^{3}-b^{5}\right)\cos\left(x\right)^{2}-3\left(a^{5}-3a^{3}b^{2}+2ab^{4}\right)\cos\left(x\right)}{3\left(a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}-\left(a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}\right)\cos\left(x\right)^{2}\right)\sin\left(x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^4*b - 10*a^2*b^3 + 8*b^5 + 2*(2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos(x)
^3 + 3*(b^4*cos(x)^2 - b^4)*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b
^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2
*cos(x)^2 + 2*a*b*cos(x) + a^2))*sin(x) + 6*(a^2*b^3 - b^5)*cos(x)^2 - 6*(a
^5 - 3*a^3*b^2 + 2*a*b^4)*cos(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - (a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(x)^2)*sin(x)), 1/3*(a^4*b - 5*a^2*b^3
+ 4*b^5 + (2*a^5 - 7*a^3*b^2 + 5*a*b^4)*cos(x)^3 - 3*(b^4*cos(x)^2 - b^4)*s
qrt(a^2 - b^2)*arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x)))*sin(x) + 3*
(a^2*b^3 - b^5)*cos(x)^2 - 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cos(x))/((a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(x)^2)*s
in(x))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(x)}{a + b\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+b*cos(x)),x)
```

```
[Out] Integral(csc(x)**4/(a + b*cos(x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(97) = 194.
time = 0.44, size = 206, normalized size = 1.87

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)b^4}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{a^2\tan\left(\frac{1}{2}x\right)^3-2ab\tan\left(\frac{1}{2}x\right)^3+b^2\tan\left(\frac{1}{2}x\right)^3+9a^2\tan\left(\frac{1}{2}x\right)-24ab\tan\left(\frac{1}{2}x\right)+15b^2\tan\left(\frac{1}{2}x\right)}{24(a^3-3a^2b+3ab^2-b^3)}-\frac{9a\tan\left(\frac{1}{2}x\right)^2+15b\tan\left(\frac{1}{2}x\right)^2+a+b}{24(a^2+2ab+b^2)\tan\left(\frac{1}{2}x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x))/sqrt(a^2 - b^2)))*b^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) +
 1/24*(a^2*tan(1/2*x)^3 - 2*a*b*tan(1/2*x)^3 + b^2*tan(1/2*x)^3 + 9*a^2*tan
(1/2*x) - 24*a*b*tan(1/2*x) + 15*b^2*tan(1/2*x))/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) - 1/24*(9*a*tan(1/2*x)^2 + 15*b*tan(1/2*x)^2 + a + b)/((a^2 + 2*a*b +
 b^2)*tan(1/2*x)^3)
```

Mupad [**B**]
time = 0.56, size = 184, normalized size = 1.67

$$\tan\left(\frac{x}{2}\right)\left(\frac{4}{8a-8b}-\frac{8a+8b}{(8a-8b)^2}\right)+\frac{\tan\left(\frac{x}{2}\right)^3}{3\,(8a-8b)}-\frac{\frac{a^2-2ab+b^2}{3(a+b)}-\frac{\tan\left(\frac{x}{2}\right)^2(-3a^3+a^2b+7ab^2-5b^3)}{(a+b)^2}}{\tan\left(\frac{x}{2}\right)^3(8a^2-16ab+8b^2)}+\frac{2b^4\,\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{3/2}}\right)}{(a+b)^{5/2}(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + b*cos(x))),x)
```

```
[Out] tan(x/2)*(4/(8*a - 8*b) - (8*a + 8*b)/(8*a - 8*b)^2) + tan(x/2)^3/(3*(8*a -
 8*b)) - ((a^2 - 2*a*b + b^2)/(3*(a + b)) - (tan(x/2)^2*(7*a*b^2 + a^2*b -
3*a^3 - 5*b^3))/(a + b)^2)/(tan(x/2)^3*(8*a^2 - 16*a*b + 8*b^2)) + (2*b^4*a
tan((tan(x/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/((a
+ b)^(5/2)*(a - b)^(5/2))
```

3.33 $\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=129

$$\frac{10ae^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}}-\frac{10ae^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d}-\frac{2ae\cos(c+dx)(e\sin(c+d}{7d}$$

```
[Out] -2/7*a*e*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/d+2/9*b*(e*sin(d*x+c))^(9/2)/d/e-1
0/21*a*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*sin(d*x+c)
)^(1/2)-10/21*a*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2748, 2715, 2721, 2720}

$$\frac{10ae^4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21d\sqrt{e\sin(c+dx)}}-\frac{10ae^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d}-\frac{2ae\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d}+\frac{2b(e\sin(c+dx))^{9/2}}{9de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (10*a*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*d*Sqrt[e
*Sin[c + d*x]]) - (10*a*e^3*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(21*d) - (2*
a*e*Cos[c + d*x]*(e*Sin[c + d*x])^(5/2))/(7*d) + (2*b*(e*Sin[c + d*x])^(9/2
))/(9*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (a+b\cos(c+dx))(e\sin(c+dx))^{7/2}\,dx = \frac{2b(e\sin(c+dx))^{9/2}}{9de} + a\int (e\sin(c+dx))^{7/2}\,dx$$

$$= -\frac{2ae\cos(c+dx)(e\sin(c+dx))^{5/2}}{7d} + \frac{2b(e\sin(c+dx))^{9/2}}{9de} + \frac{1}{7}(5$$

$$= -\frac{10ae^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} - \frac{2ae\cos(c+dx)(e\sin(c+}{7d}$$

$$= -\frac{10ae^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} - \frac{2ae\cos(c+dx)(e\sin(c+}{7d}$$

$$= \frac{10ae^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}} - \frac{10ae^3\cos(c+dx)\sqrt{}}{21d}$$

Mathematica [A]
time = 0.91, size = 108, normalized size = 0.84

$$\frac{e^3\left(-120aF\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+(21b-138a\cos(c+dx)-28b\cos(2(c+dx))+18a\cos(3(c+dx))+7b\cos(4(c+dx)))\sqrt{\sin(c+dx)}\right)\sqrt{e\sin(c+dx)}}{252d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (e^3*(-120*a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (21*b - 138*a*Cos[c + d*
x] - 28*b*Cos[2*(c + d*x)] + 18*a*Cos[3*(c + d*x)] + 7*b*Cos[4*(c + d*x)])*
Sqrt[Sin[c + d*x]])*Sqrt[e*Sin[c + d*x]])/(252*d*Sqrt[Sin[c + d*x]])
```

Maple [A]
time = 0.14, size = 127, normalized size = 0.98

method	result

default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{9}{2}}}{9e}-\frac{e^4a\left(-6\left(\sin^5(dx+c)\right)+5\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\text{EllipticF}\left(\sqrt{-\right.\right.}{21\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/9/e*b*(e*sin(d*x+c))^(9/2)-1/21*e^4*a*(-6*sin(d*x+c)^5+5*(-sin(d*x+c)+1)
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1
/2),1/2*2^(1/2))-4*sin(d*x+c)^3+10*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(1
/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((b*cos(d*x + c) + a)*sin(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 130, normalized size = 1.01

$$\frac{15\sqrt{2}\sqrt{-i}\,ae^{\frac{7}{2}}\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+15\sqrt{2}\sqrt{i}\,ae^{\frac{7}{2}}\text{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))+2\left(7b\cos(dx+c)^4e^{\frac{7}{2}}+9a\cos(dx+c)^3e^{\frac{7}{2}}-14b\cos(dx+c)^2e^{\frac{7}{2}}-24a\cos(dx+c)e^{\frac{7}{2}}+7be^{\frac{7}{2}}\right)\sqrt{\sin(dx+c)}}{63d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/63*(15*sqrt(2)*sqrt(-I)*a*e^(7/2)*weierstrassPInverse(4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 15*sqrt(2)*sqrt(I)*a*e^(7/2)*weierstrassPInverse(4, 0,
cos(d*x + c) - I*sin(d*x + c)) + 2*(7*b*cos(d*x + c)^4*e^(7/2) + 9*a*cos(d*
x + c)^3*e^(7/2) - 14*b*cos(d*x + c)^2*e^(7/2) - 24*a*cos(d*x + c)*e^(7/2)
+ 7*b*e^(7/2))*sqrt(sin(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4846 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)*e^(7/2)*sin(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\sin\left(c+d\,x\right))^{7/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x)),x)
```

```
[Out] int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x)), x)
```

3.34 $\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=100

$$\frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

```
[Out] -2/5*a*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+2/7*b*(e*sin(d*x+c))^(7/2)/d/e-6
/5*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elli
pticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+c)^
(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2748, 2715, 2721, 2719}

$$\frac{6ae^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}+\frac{2b(e\sin(c+dx))^{7/2}}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (6*a*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[S
in[c + d*x]]) - (2*a*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d) + (2*b*(e
*Sin[c + d*x])^(7/2))/(7*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))(e\sin(c+dx))^{5/2}\,dx &= \frac{2b(e\sin(c+dx))^{7/2}}{7de}+a\int (e\sin(c+dx))^{5/2}\,dx\\ &= -\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}+\frac{2b(e\sin(c+dx))^{7/2}}{7de}+\frac{1}{5}(3\\ &= -\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}+\frac{2b(e\sin(c+dx))^{7/2}}{7de}+\frac{(3a}{}\\ &= \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2ae\cos(c+dx)(e\,\mathrm{s}}{5d}\end{aligned}$$

Mathematica [A]
time = 0.55, size = 80, normalized size = 0.80

$$\frac{2(e\sin(c+dx))^{5/2}\left(-21aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+\sin^{\frac{3}{2}}(c+dx)\left(-7a\cos(c+dx)+5b\sin^2(c+dx)\right)\right)}{35d\sin^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(e*Sin[c + d*x])^(5/2)*(-21*a*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + Sin[
c + d*x]^(3/2)*(-7*a*Cos[c + d*x] + 5*b*Sin[c + d*x]^2)))/(35*d*Sin[c + d*x
]^(5/2))
```

Maple [A]
time = 0.13, size = 171, normalized size = 1.71

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{7}{2}}}{7e}-\frac{e^3a\left(6\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)}\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/7/e*b*(e*sin(d*x+c))^(7/2)-1/5*e^3*a*(6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-
3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^4+2*sin(d*x+c)^2)/cos(d*x+c)
/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((b*cos(d*x + c) + a)*sin(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 110, normalized size = 1.10

$$\frac{21i\sqrt{2}\,\sqrt{-i}\,ae^{\frac{5}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)))-21i\sqrt{2}\,\sqrt{i}\,ae^{\frac{5}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c)))-2\left(5\,b\cos(dx+c)^2\,e^{\frac{5}{2}}+7\,a\cos(dx+c)\,e^{\frac{5}{2}}-5\,be^{\frac{5}{2}}\right)\sin(dx+c)^{\frac{3}{2}}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/35*(21*I*sqrt(2)*sqrt(-I)*a*e^(5/2)*weierstrassZeta(4, 0, weierstrassPInv
erse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(I)*a*e^(5/2)
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
 c))) - 2*(5*b*cos(d*x + c)^2*e^(5/2) + 7*a*cos(d*x + c)*e^(5/2) - 5*b*e^(5
/2))*sin(d*x + c)^(3/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sin(c+dx))^{\frac{5}{2}}\,(a+b\cos(c+dx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((e*sin(c + d*x))**(5/2)*(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)*e^(5/2)*sin(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x)),x)
```

```
[Out] int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x)), x)
```

3.35 $\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=100

$$\frac{2ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}}-\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

```
[Out] 2/5*b*(e*sin(d*x+c))^(5/2)/d/e-2/3*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*si
n(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)-2/3*a*e*cos(d*x+c)*(e*sin(d*x+c))^(1/
2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2748, 2715, 2721, 2720}

$$\frac{2ae^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}+\frac{2b(e\sin(c+dx))^{5/2}}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*a*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*S
in[c + d*x]]) - (2*a*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d) + (2*b*(e*S
in[c + d*x])^(5/2))/(5*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+b\cos(c+dx))(e\sin(c+dx))^{3/2}\,dx &= \frac{2b(e\sin(c+dx))^{5/2}}{5de}+a\int (e\sin(c+dx))^{3/2}\,dx \\
&= -\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}+\frac{2b(e\sin(c+dx))^{5/2}}{5de}+\frac{1}{3}(ae \\
&= -\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}+\frac{2b(e\sin(c+dx))^{5/2}}{5de}+\frac{\left(ae^2\right.}{} \\
&= \frac{2ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}}-\frac{2ae\cos(c+dx)\sqrt{e\,s}}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.50, size = 80, normalized size = 0.80

$$\frac{2(e\sin(c+dx))^{3/2}\left(-5aF\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)+\sqrt{\sin(c+dx)}\,(-5a\cos(c+dx)+3b\sin^2(c+dx))\right)}{15d\sin^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(e*Sin[c + d*x])^(3/2)*(-5*a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + Sqrt[
Sin[c + d*x]]*(-5*a*Cos[c + d*x] + 3*b*Sin[c + d*x]^2)))/(15*d*Sin[c + d*x]
^(3/2))
```

Maple [A]
time = 0.13, size = 116, normalized size = 1.16

method	result
default	$\dfrac{\frac{2b(e\sin(dx+c))^{\frac{5}{2}}}{5e}-\dfrac{e^2a\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+}\right.\right.}{3\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/5/e*b*(e*sin(d*x+c))^(5/2)-1/3*e^2*a*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*
sin(d*x+c)^3+2*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((b*cos(d*x + c) + a)*sin(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 104, normalized size = 1.04

$$\frac{5\sqrt{2}\sqrt{-i}\,ae^{\frac{3}{2}}\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}\sqrt{i}\,ae^{\frac{3}{2}}\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))-2\left(3\,b\cos(dx+c)^{2}e^{\frac{3}{2}}+5\,a\cos(dx+c)\,e^{\frac{3}{2}}-3\,be^{\frac{3}{2}}\right)\sqrt{\sin(dx+c)}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(5*sqrt(2)*sqrt(-I)*a*e^(3/2)*weierstrassPInverse(4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 5*sqrt(2)*sqrt(I)*a*e^(3/2)*weierstrassPInverse(4, 0, co
s(d*x + c) - I*sin(d*x + c)) - 2*(3*b*cos(d*x + c)^2*e^(3/2) + 5*a*cos(d*x
+ c)*e^(3/2) - 3*b*e^(3/2))*sqrt(sin(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sin(c+dx))^{\frac{3}{2}}\,(a+b\cos(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)*e^(3/2)*sin(d*x + c)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (e \sin(c+dx))^{3/2} (a+b\cos(c+dx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x)), x)
```

3.36 $\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx$

Optimal. Leaf size=68

$$\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

```
[Out] 2/3*b*(e*sin(d*x+c))^(3/2)/d/e-2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(
1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d
*x+c))^(1/2)/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2748, 2721, 2719}

$$\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c +
 d*x]]) + (2*b*(e*Sin[c + d*x])^(3/2))/(3*d*e)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (a+b\cos(c+dx))\sqrt{e\sin(c+dx)}\,dx = \frac{2b(e\sin(c+dx))^{3/2}}{3de} + a\int \sqrt{e\sin(c+dx)}\,dx$$
$$= \frac{2b(e\sin(c+dx))^{3/2}}{3de} + \frac{\left(a\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{\sqrt{\sin(c+dx)}}$$
$$= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{2b(e\sin(c+dx))^{3/2}}{3de}$$

Mathematica [A]
time = 0.11, size = 60, normalized size = 0.88

$$\frac{2\sqrt{e\sin(c+dx)}\left(-3aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+b\sin^{\frac{3}{2}}(c+dx)\right)}{3d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[e*Sin[c + d*x]]*(-3*a*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + b*Sin[c
 + d*x]^(3/2)))/(3*d*Sqrt[Sin[c + d*x]])
```

Maple [A]
time = 0.13, size = 117, normalized size = 1.72

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{3}{2}}}{3e}-\frac{ae\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\left(2\,\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+}\right.\right.}{\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/3*b/e*(e*sin(d*x+c))^(3/2)-a*e*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)*(2*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-Ellipt
icF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*cos(d*x + c) + a)*sqrt(sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 83, normalized size = 1.22

$$\frac{3i\sqrt{2}\sqrt{-i}\,ae^{\frac{1}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\sqrt{i}\,ae^{\frac{1}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))+2\,be^{\frac{1}{2}}\sin(dx+c)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*I*sqrt(2)*sqrt(-I)*a*e^(1/2)*weierstrassZeta(4, 0, weierstrassPInver
se(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*sqrt(I)*a*e^(1/2)*we
ierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)
)) + 2*b*e^(1/2)*sin(d*x + c)^(3/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{e\sin(c+dx)}\,(a+b\cos(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)*e^(1/2)*sqrt(sin(d*x + c)), x)
```

Mupad [B]
time = 0.49, size = 60, normalized size = 0.88

$$\frac{2\,b\,\sin(c+d\,x)\,\sqrt{e\,\sin(c+d\,x)}}{3\,d}+\frac{2\,a\,\sqrt{e\,\sin(c+d\,x)}\,\mathrm{E}\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{\sin(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x)),x)
```

```
[Out] (2*b*sin(c + d*x)*(e*sin(c + d*x))^(1/2))/(3*d) + (2*a*(e*sin(c + d*x))^(1/
2)*ellipticE(c/2 - pi/4 + (d*x)/2, 2))/(d*sin(c + d*x)^(1/2))
```

3.37 $\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=66

$$\frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}+\frac{2b\sqrt{e\sin(c+dx)}}{de}$$

```
[Out] -2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
F(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2
)+2*b*(e*sin(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2748, 2721, 2720}

$$\frac{2a\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{e\sin(c+dx)}}+\frac{2b\sqrt{e\sin(c+dx)}}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c +
 d*x]]) + (2*b*Sqrt[e*Sin[c + d*x]])/(d*e)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \frac{a+b\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2b\sqrt{e\sin(c+dx)}}{de} + a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{2b\sqrt{e\sin(c+dx)}}{de} + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{\sqrt{e\sin(c+dx)}}$$
$$= \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{2b\sqrt{e\sin(c+dx)}}{de}$$

Mathematica [A]
time = 0.22, size = 54, normalized size = 0.82

$$\frac{2\left(-aF\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sqrt{\sin(c+dx)}+b\sin(c+dx)\right)}{d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
[Out] (2*(-(a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]) + b*Sin[c +
 d*x]))/(d*Sqrt[e*Sin[c + d*x]])
```

Maple [A]
time = 0.20, size = 92, normalized size = 1.39

method	result
default	$-\frac{a\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-}{\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$
risch	$-\frac{ib(\mathrm{e}^{2i(dx+c)}-1)\sqrt{2}\,\mathrm{e}^{-i(dx+c)}}{d\sqrt{-ie\left(\mathrm{e}^{2i(dx+c)}-1\right)\mathrm{e}^{-i(dx+c)}}}-\frac{ia\sqrt{\mathrm{e}^{i(dx+c)}+1}\,\sqrt{-2\,\mathrm{e}^{i(dx+c)}+2}\,\sqrt{-\mathrm{e}^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{\mathrm{e}^{i(dx}\right.}{d\sqrt{-ie\,\mathrm{e}^{3i(dx+c)}+ie\,\mathrm{e}^{i(dx+c)}}\,\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*sin
(d*x+c)*cos(d*x+c)*b)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*cos(d*x + c) + a)/sqrt(sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 70, normalized size = 1.06

$$\frac{\left(\sqrt{2}\,\sqrt{-i}\,a\mathrm{weierstrassPInverse}\left(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\,\sqrt{i}\,a\mathrm{weierstrassPInverse}\left(4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+2\,b\sqrt{\sin\left(dx+c\right)}\right)e^{\left(-\frac{1}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*sqrt(-I)*a*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)
) + sqrt(2)*sqrt(I)*a*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
c)) + 2*b*sqrt(sin(d*x + c)))*e^(-1/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\cos\left(c+dx\right)}{\sqrt{e\sin\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))/sqrt(e*sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)*e^(-1/2)/sqrt(sin(d*x + c)), x)
```

Mupad [B]
time = 0.72, size = 50, normalized size = 0.76

$$-\frac{2\,\sqrt{\sin\left(c+d\,x\right)}\,\left(a\,\mathrm{F}\left(\frac{\pi}{4}-\frac{c}{2}-\frac{d\,x}{2}\middle|2\right)-b\,\sqrt{\sin\left(c+d\,x\right)}\right)}{d\,\sqrt{e\,\sin\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(1/2),x)
```

```
[Out] -(2*sin(c + d*x)^(1/2)*(a*ellipticF(pi/4 - c/2 - (d*x)/2, 2) - b*sin(c + d*
x)^(1/2)))/(d*(e*sin(c + d*x))^(1/2))
```

3.38 $\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{2b}{de\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}$$

```
[Out] -2*b/d/e/(e*sin(d*x+c))^(1/2)-2*a*cos(d*x+c)/d/e/(e*sin(d*x+c))^(1/2)+2*a*(
sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(
1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2748, 2716, 2721, 2719}

$$-\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{2b}{de\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*b)/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c +
d*x]]) - (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*e^2
*Sqrt[Sin[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx &= -\frac{2b}{de\sqrt{e\sin(c+dx)}}+a\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx\\ &= -\frac{2b}{de\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{a\int \sqrt{e\sin(c+dx)}\,dx}{e^2}\\ &= -\frac{2b}{de\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{\left(a\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}}{e^2\sqrt{\sin(c+dx)}}\\ &= -\frac{2b}{de\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+}}{de^2\sqrt{\sin(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 58, normalized size = 0.60

$$-\frac{2\left(b+a\cos(c+dx)-aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
[Out] (-2*(b + a*Cos[c + d*x] - a*EllipticE[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c
+ d*x]]))/(d*e*Sqrt[e*Sin[c + d*x]])
```

Maple [A]
time = 0.12, size = 153, normalized size = 1.59

method	result
default	$\frac{2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)a-a\sqrt{}}{e\cos(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a-a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*a*
cos(d*x+c)^2-2*b*cos(d*x+c))/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((b*cos(d*x + c) + a)/sin(d*x + c)^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 107, normalized size = 1.11

$$\frac{\left(-i\sqrt{2}\sqrt{-i}\,a\sin\left(dx+c\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+i\sqrt{2}\sqrt{i}\,a\sin\left(dx+c\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-2\left(a\cos\left(dx+c\right)+b\right)\sqrt{\sin\left(dx+c\right)}\right)e^{\left(-\frac{3}{2}\right)}}{d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(-I)*a*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInver
se(4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(I)*a*sin(d*x + c)
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
 c))) - 2*(a*cos(d*x + c) + b)*sqrt(sin(d*x + c)))*e^(-3/2)/(d*sin(d*x + c)
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\cos\left(c + dx\right)}{\left(e\sin\left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))/(e*sin(c + d*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)*e^(-3/2)/sin(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\cos\left(c+d\,x\right)}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(3/2),x)
[Out] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(3/2), x)
```

3.39 $\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{2b}{3de(e\sin(c+dx))^{3/2}}-\frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}+\frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}}$$

```
[Out] -2/3*b/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*cos(d*x+c)/d/e/(e*sin(d*x+c))^(3/2)-2
/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
F(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^2/(e*sin(d*x+c))^
(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2748, 2716, 2721, 2720}

$$\frac{2a\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}-\frac{2b}{3de(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*b)/(3*d*e*(e*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x])/(3*d*e*(e*Sin[c
+ d*x])^(3/2)) + (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/
(3*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2b}{3de(e\sin(c+dx))^{3/2}} + a\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx$$
$$= -\frac{2b}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2}$$
$$= -\frac{2b}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+}}}{3e^2\sqrt{e\sin(c+dx)}}$$
$$= -\frac{2b}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c}}{3de^2\sqrt{e\sin(c+dx)}}$$

Mathematica [A]
time = 0.17, size = 59, normalized size = 0.58

$$-\frac{2\left(b+a\cos(c+dx)+aF\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3de(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x] + a*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x
]^(3/2)))/(3*d*e*(e*Sin[c + d*x])^(3/2))
```

Maple [A]
time = 0.13, size = 124, normalized size = 1.22

method	result
default	$\frac{-\frac{2b}{3e(e\sin(dx+c))^{\frac{3}{2}}}-\frac{a\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1}\right.\right.}{3e^2\sin(dx+c)^2\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/3*b/e/(e*sin(d*x+c))^(3/2)-1/3*a/e^2*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2
*sin(d*x+c)^3+2*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((b*cos(d*x + c) + a)/sin(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 127, normalized size = 1.25

$$\frac{\sqrt{-i}\left(\sqrt{2}\,a\cos\left(dx+c\right)^{2}-\sqrt{2}\,a\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{i}\left(\sqrt{2}\,a\cos\left(dx+c\right)^{2}-\sqrt{2}\,a\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\left(a\cos\left(dx+c\right)+b\right)\sqrt{\sin\left(dx+c\right)}}{3\left(d\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}-de^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(-I)*(sqrt(2)*a*cos(d*x + c)^2 - sqrt(2)*a)*weierstrassPInverse(4,
 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(I)*(sqrt(2)*a*cos(d*x + c)^2 - sq
rt(2)*a)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(a*co
s(d*x + c) + b)*sqrt(sin(d*x + c)))/(d*cos(d*x + c)^2*e^(5/2) - d*e^(5/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\cos\left(c + dx\right)}{\left(e\sin\left(c + dx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))/(e*sin(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)*e^(-5/2)/sin(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\cos\left(c+d\,x\right)}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(5/2), x)
```

3.40 $\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{2b}{5de(e\sin(c+dx))^{5/2}}-\frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}-\frac{6a\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}}-\frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}}$$

```
[Out] -2/5*b/d/e/(e*sin(d*x+c))^(5/2)-2/5*a*cos(d*x+c)/d/e/(e*sin(d*x+c))^(5/2)-6
/5*a*cos(d*x+c)/d/e^3/(e*sin(d*x+c))^(1/2)+6/5*a*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(
1/2))*(e*sin(d*x+c))^(1/2)/d/e^4/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2748, 2716, 2721, 2719}

$$-\frac{6aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}}-\frac{6a\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}}-\frac{2b}{5de(e\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*b)/(5*d*e*(e*Sin[c + d*x])^(5/2)) - (2*a*Cos[c + d*x])/(5*d*e*(e*Sin[c
+ d*x])^(5/2)) - (6*a*Cos[c + d*x])/(5*d*e^3*Sqrt[e*Sin[c + d*x]]) - (6*a*E
llipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*e^4*Sqrt[Sin[c +
 d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx &= -\frac{2b}{5de(e\sin(c+dx))^{5/2}} + a\int \frac{1}{(e\sin(c+dx))^{7/2}}\,dx \\
&= -\frac{2b}{5de(e\sin(c+dx))^{5/2}} - \frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}} + \frac{(3a)\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{5e^2} \\
&= -\frac{2b}{5de(e\sin(c+dx))^{5/2}} - \frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}} - \frac{6a\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{(3a)\int \sqrt{\ldots}}{\ldots} \\
&= -\frac{2b}{5de(e\sin(c+dx))^{5/2}} - \frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}} - \frac{6a\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{\left(3a\sqrt{e \ldots}\right.}{\ldots} \\
&= -\frac{2b}{5de(e\sin(c+dx))^{5/2}} - \frac{2a\cos(c+dx)}{5de(e\sin(c+dx))^{5/2}} - \frac{6a\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}} - \frac{6aE\left(\frac{1}{2}\right. \ldots}{\ldots}
\end{aligned}
$$

Mathematica [A]
time = 0.28, size = 74, normalized size = 0.56

$$
\frac{-4b-7a\cos(c+dx)+3a\cos(3(c+dx))+12aE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{5}{2}}(c+dx)}{10de(e\sin(c+dx))^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])/(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-4*b - 7*a*Cos[c + d*x] + 3*a*Cos[3*(c + d*x)] + 12*a*EllipticE[(-2*c + Pi
 - 2*d*x)/4, 2]*Sin[c + d*x]^(5/2))/(10*d*e*(e*Sin[c + d*x])^(5/2))
```

Maple [A]
time = 0.14, size = 187, normalized size = 1.43

method	result
default	$-\frac{2b}{5e(e\sin(dx+c))^{\frac{5}{2}}}+\frac{a\left(6\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1 \ldots}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/5*b/e/(e*sin(d*x+c))^(5/2)+1/5*a/e^3*(6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))
-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+6*sin(d*x+c)^5-4*sin(d*x+c)^3-2*sin(d*x+
c))/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((b*cos(d*x + c) + a)/sin(d*x + c)^(7/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 171, normalized size = 1.31

$$-\frac{3\sqrt{-i}\left(i\sqrt{2}\,a\cos\left(dx+c\right)^{2}-i\sqrt{2}\,a\right)\sin\left(dx+c\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+3\sqrt{i}\left(-i\sqrt{2}\,a\cos\left(dx+c\right)^{2}+i\sqrt{2}\,a\right)\sin\left(dx+c\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\left(3\,a\cos\left(dx+c\right)^{3}-4\,a\cos\left(dx+c\right)-b\right)\sqrt{\sin\left(dx+c\right)}}{5\left(d\cos\left(dx+c\right)^{2}e^{\frac{7}{2}}-de^{\frac{7}{2}}\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*sqrt(-I)*(I*sqrt(2)*a*cos(d*x + c)^2 - I*sqrt(2)*a)*sin(d*x + c)*we
ierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + 3*sqrt(I)*(-I*sqrt(2)*a*cos(d*x + c)^2 + I*sqrt(2)*a)*sin(d*x + c)*wei
erstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))
) + 2*(3*a*cos(d*x + c)^3 - 4*a*cos(d*x + c) - b)*sqrt(sin(d*x + c)))/((d*c
os(d*x + c)^2*e^(7/2) - d*e^(7/2))*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)*e^(-7/2)/sin(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\cos\left(c+d\,x\right)}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(7/2),x)
[Out] int((a + b*cos(c + d*x))/(e*sin(c + d*x))^(7/2), x)
```

3.41 $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=193

$$\frac{10(11a^2+2b^2)\,e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{231d\sqrt{e\sin(c+dx)}}-\frac{10(11a^2+2b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d}-\frac{2(1\ldots}{}$$

```
[Out] -2/77*(11*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/d+26/99*a*b*(e*sin(d
*x+c))^(9/2)/d/e+2/11*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(9/2)/d/e-10/231*(1
1*a^2+2*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*sin(
d*x+c))^(1/2)-10/231*(11*a^2+2*b^2)*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.13, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2771, 2748, 2715, 2721, 2720}

$$\frac{10e^4(11a^2+2b^2)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)}{231d\sqrt{e\sin(c+dx)}}-\frac{10e^3(11a^2+2b^2)\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d}-\frac{2e(11a^2+2b^2)\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d}+\frac{26ab(e\sin(c+dx))^{9/2}}{99de}+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{11de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (10*(11*a^2 + 2*b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/(231*d*Sqrt[e*Sin[c + d*x]]) - (10*(11*a^2 + 2*b^2)*e^3*Cos[c + d*x]*Sqr
t[e*Sin[c + d*x]])/(231*d) - (2*(11*a^2 + 2*b^2)*e*Cos[c + d*x]*(e*Sin[c +
d*x])^(5/2))/(77*d) + (26*a*b*(e*Sin[c + d*x])^(9/2))/(99*d*e) + (2*b*(a +
b*Cos[c + d*x])*(e*Sin[c + d*x])^(9/2))/(11*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}\,dx = \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{11de}+\frac{2}{11}\int\left(\frac{11a^2}{2}+b^2+\frac{1}{}\right.$$
$$=\frac{26ab(e\sin(c+dx))^{9/2}}{99de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{11de}$$
$$=-\frac{2(11a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d}+\frac{26ab(e\sin(c+}{99de}$$
$$=-\frac{10(11a^2+2b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d}-\frac{2(11a^2+2b^2}{}$$
$$=-\frac{10(11a^2+2b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d}-\frac{2(11a^2+2b^2}{}$$
$$=\frac{10(11a^2+2b^2)\,e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{231d\sqrt{e\sin(c+dx)}}-\frac{10(11a}{}$$

Mathematica [A]
time = 1.73, size = 157, normalized size = 0.81

$$\frac{\left(\frac{1}{6}(924ab-6(506a^2+71b^2)\cos(c+dx)-1232ab\cos(2(c+dx))+396a^2\cos(3(c+dx))-117b^2\cos(3(c+dx))+308ab\cos(4(c+dx))+63b^2\cos(5(c+dx)))\csc^3(c+dx)-\frac{40(11a^2+2b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)|2\right)}{\sin^{\frac{7}{2}}(c+dx)}\right)(e\sin(c+dx))^{7/2}}{924d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] ((((924*a*b - 6*(506*a^2 + 71*b^2)*Cos[c + d*x] - 1232*a*b*Cos[2*(c + d*x)]
 + 396*a^2*Cos[3*(c + d*x)] - 117*b^2*Cos[3*(c + d*x)] + 308*a*b*Cos[4*(c +
 d*x)] + 63*b^2*Cos[5*(c + d*x)])*Csc[c + d*x]^3)/6 - (40*(11*a^2 + 2*b^2)*
EllipticF[(-2*c + Pi - 2*d*x)/4, 2])/Sin[c + d*x]^(7/2))*(e*Sin[c + d*x])^(
7/2))/(924*d)
```

Maple [A]
time = 0.14, size = 252, normalized size = 1.31

method	result
default	$\frac{\frac{4ab(e\sin(dx+c))^{\frac{9}{2}}}{9e} - \frac{e^4\left(-42b^2\left(\cos^6(dx+c)\right)\sin(dx+c)-66a^2\left(\cos^4(dx+c)\right)\sin(dx+c)+72b^2\left(\cos^4(dx+c)\right)\sin(dx+c)+55\sqrt{-\sin(dx+c)}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4/9/e*a*b*(e*sin(d*x+c))^(9/2)-1/231*e^4*(-42*b^2*cos(d*x+c)^6*sin(d*x+c)-
66*a^2*cos(d*x+c)^4*sin(d*x+c)+72*b^2*cos(d*x+c)^4*sin(d*x+c)+55*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))*a^2+10*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2+176*a^2*co
s(d*x+c)^2*sin(d*x+c)-10*b^2*cos(d*x+c)^2*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x
+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((b*cos(d*x + c) + a)^2*sin(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 188, normalized size = 0.97

$$\frac{15\sqrt{2}\sqrt{-i}\,(11a^2+2b^2)e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+15\sqrt{2}\sqrt{i}\,(11a^2+2b^2)e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))+2\left(63b^2\cos(dx+c)^5e^{\frac{7}{2}}+154ab\cos(dx+c)^4e^{\frac{7}{2}}-308ab\cos(dx+c)^2e^{\frac{7}{2}}+9(11a^2-12b^2)\cos(dx+c)^3e^{\frac{7}{2}}+154abe^{\frac{7}{2}}-3(88a^2-5b^2)\cos(dx+c)e^{\frac{7}{2}}\right)\sqrt{\sin(dx+c)}}{693d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/693*(15*sqrt(2)*sqrt(-I)*(11*a^2 + 2*b^2)*e^(7/2)*weierstrassPInverse(4,
0, cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*sqrt(I)*(11*a^2 + 2*b^2)*e^(
```

```
7/2)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(63*b^2*c
os(d*x + c)^5*e^(7/2) + 154*a*b*cos(d*x + c)^4*e^(7/2) - 308*a*b*cos(d*x +
c)^2*e^(7/2) + 9*(11*a^2 - 12*b^2)*cos(d*x + c)^3*e^(7/2) + 154*a*b*e^(7/2)
 - 3*(88*a^2 - 5*b^2)*cos(d*x + c)*e^(7/2))*sqrt(sin(d*x + c)))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5986 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2*e^(7/2)*sin(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e \sin (c + d\,x))^{7/2}\,(a + b\,\cos (c + d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2, x)
```

3.42 $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=154

$$\frac{2(9a^2+2b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{15d\sqrt{\sin(c+dx)}}-\frac{2(9a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{45d}+\frac{22ab(e$$

```
[Out] -2/45*(9*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+22/63*a*b*(e*sin(d*
x+c))^(7/2)/d/e+2/9*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(7/2)/d/e-2/15*(9*a^2
+2*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+
c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2771, 2748, 2715, 2721, 2719}

$$\frac{2e^2(9a^2+2b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{15d\sqrt{\sin(c+dx)}}-\frac{2e(9a^2+2b^2)\cos(c+dx)(e\sin(c+dx))^{3/2}}{45d}+\frac{22ab(e\sin(c+dx))^{7/2}}{63de}+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{9de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(9*a^2 + 2*b^2)*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]
])/(15*d*Sqrt[Sin[c + d*x]]) - (2*(9*a^2 + 2*b^2)*e*Cos[c + d*x]*(e*Sin[c +
 d*x])^(3/2))/(45*d) + (22*a*b*(e*Sin[c + d*x])^(7/2))/(63*d*e) + (2*b*(a +
 b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2))/(9*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}\,dx &= \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{9de}+\frac{2}{9}\int\left(\frac{9a^2}{2}+b^2+\frac{11}{2}\right.\\ &= \frac{22ab(e\sin(c+dx))^{7/2}}{63de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{9de}\\ &= -\frac{2(9a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{45d}+\frac{22ab(e\sin(c+}{63de}\\ &= -\frac{2(9a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{45d}+\frac{22ab(e\sin(c+}{63de}\\ &= \frac{2(9a^2+2b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15d\sqrt{\sin(c+dx)}}-\frac{2(9a^2+}{}\end{aligned}$$

Mathematica [A]
time = 0.84, size = 116, normalized size = 0.75

$$-\frac{(e\sin(c+dx))^{5/2}\left(84(9a^2+2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+(21(12a^2+b^2)\cos(c+dx)+5b(-36a+36a\cos(2(c+dx))+7b\cos(3(c+dx))))\sin^{\frac{3}{2}}(c+dx)\right)}{630d\sin^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/630*((e*Sin[c + d*x])^(5/2)*(84*(9*a^2 + 2*b^2)*EllipticE[(-2*c + Pi - 2
*d*x)/4, 2] + (21*(12*a^2 + b^2)*Cos[c + d*x] + 5*b*(-36*a + 36*a*Cos[2*(c
```

```
+ d*x)] + 7*b*Cos[3*(c + d*x)]))*Sin[c + d*x]^(3/2)))/(d*Sin[c + d*x]^(5/2)
)
```

Maple [A]
time = 0.15, size = 332, normalized size = 2.16

method	result
default	$\frac{\frac{4ab(e\sin(dx+c))^{\frac{7}{2}}}{7e}-\frac{e^3\left(10b^2\left(\sin^6(dx+c)\right)+54\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\text{EllipticE}\left(\sqrt{\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4/7/e*a*b*(e*sin(d*x+c))^(7/2)-1/45*e^3*(10*b^2*sin(d*x+c)^6+54*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))*a^2+12*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-27*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))*a^2-6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-18*a^2*s
in(d*x+c)^4-14*sin(d*x+c)^4*b^2+18*sin(d*x+c)^2*a^2+4*sin(d*x+c)^2*b^2)/cos
(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((b*cos(d*x + c) + a)^2*sin(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 157, normalized size = 1.02

$$\frac{21i\sqrt{2}\sqrt{-i}\,(9a^2+2b^2)e^{\frac{5}{2}}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}\sqrt{i}\,(9a^2+2b^2)e^{\frac{5}{2}}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(35b^2\cos(dx+c)^3e^{\frac{5}{2}}+90ab\cos(dx+c)^2e^{\frac{5}{2}}-90abe^{\frac{5}{2}}+21(3a^2-b^2)\cos(dx+c)e^{\frac{5}{2}}\right)\sin(dx+c)^{\frac{3}{2}}}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(21*I*sqrt(2)*sqrt(-I)*(9*a^2 + 2*b^2)*e^(5/2)*weierstrassZeta(4, 0,
weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sq
rt(I)*(9*a^2 + 2*b^2)*e^(5/2)*weierstrassZeta(4, 0, weierstrassPInverse(4,
```

```
0, cos(d*x + c) - I*sin(d*x + c))) - 2*(35*b^2*cos(d*x + c)^3*e^(5/2) + 90*
a*b*cos(d*x + c)^2*e^(5/2) - 90*a*b*e^(5/2) + 21*(3*a^2 - b^2)*cos(d*x + c)
*e^(5/2))*sin(d*x + c)^(3/2))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2*e^(5/2)*sin(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2, x)
```

3.43 $\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=154

$$\frac{2(7a^2+2b^2)\,e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}}-\frac{2(7a^2+2b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d}+\frac{18ab(e\sin}{3}$$

```
[Out] 18/35*a*b*(e*sin(d*x+c))^(5/2)/d/e+2/7*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(5
/2)/d/e-2/21*(7*a^2+2*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(
1/2)/d/(e*sin(d*x+c))^(1/2)-2/21*(7*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^
(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2771, 2748, 2715, 2721, 2720}

$$\frac{2e^2(7a^2+2b^2)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)}{21d\sqrt{e\sin(c+dx)}}-\frac{2e(7a^2+2b^2)\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d}+\frac{18ab(e\sin(c+dx))^{5/2}}{35de}+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(7*a^2 + 2*b^2)*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])
/(21*d*Sqrt[e*Sin[c + d*x]]) - (2*(7*a^2 + 2*b^2)*e*Cos[c + d*x]*Sqrt[e*Sin
[c + d*x]])/(21*d) + (18*a*b*(e*Sin[c + d*x])^(5/2))/(35*d*e) + (2*b*(a + b
*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(7*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{7de} + \frac{2}{7}\int\left(\frac{7a^2}{2}+b^2+\frac{9}{2}ab\right.$$

$$= \frac{18ab(e\sin(c+dx))^{5/2}}{35de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{7de}$$

$$= -\frac{2(7a^2+2b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} + \frac{18ab(e\sin(c+d}{35de}$$

$$= -\frac{2(7a^2+2b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d} + \frac{18ab(e\sin(c+d}{35de}$$

$$= \frac{2(7a^2+2b^2)\,e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}} - \frac{2(7a^2+}{}$$

Mathematica [A]
time = 0.87, size = 117, normalized size = 0.76

$$\frac{\left(-\frac{1}{2}(5(28a^2+5b^2)\cos(c+dx)+3b(-28a+28a\cos(2(c+dx))+5b\cos(3(c+dx))))\csc(c+dx)-\frac{10(7a^2+2b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)|2\right)}{\sin^{\frac{3}{2}}(c+dx)}\right)(e\sin(c+dx))^{3/2}}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((-1/2*((5*(28*a^2 + 5*b^2)*Cos[c + d*x] + 3*b*(-28*a + 28*a*Cos[2*(c + d*x
)] + 5*b*Cos[3*(c + d*x)]))*Csc[c + d*x]) - (10*(7*a^2 + 2*b^2)*EllipticF[(
```

```
-2*c + Pi - 2*d*x)/4, 2])/Sin[c + d*x]^(3/2))*(e*Sin[c + d*x])^(3/2))/(105*
d)
```

Maple [A]
time = 0.13, size = 229, normalized size = 1.49

method	result
default	$-\frac{e^{2}\left(30 b^{2}\left(\cos^{4}\left(d x+c\right)\right) \sin\left(d x+c\right)+35 \sqrt{-\sin\left(d x+c\right)+1}\, \sqrt{2 \sin\left(d x+c\right)+2}\, \left(\sqrt{\sin}\left(d x+c\right)\right) \mathrm{EllipticF}\left(\sqrt{\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^2*(30*b^2*cos(d*x+c)^4*sin(d*x+c)+
35*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+10*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b
^2+84*a*b*cos(d*x+c)^3*sin(d*x+c)+70*a^2*cos(d*x+c)^2*sin(d*x+c)-10*b^2*cos
(d*x+c)^2*sin(d*x+c)-84*a*b*cos(d*x+c)*sin(d*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((b*cos(d*x + c) + a)^2*sin(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 151, normalized size = 0.98

$$\frac{5 \sqrt{2} \sqrt{-i} \left(7 a^{2}+2 b^{2}\right) e^{\frac{3}{2}} \mathrm{weierstrassPInverse}\left(4,0,\cos\left(d x+c\right)+i \sin\left(d x+c\right)\right)+5 \sqrt{2} \sqrt{i} \left(7 a^{2}+2 b^{2}\right) e^{\frac{3}{2}} \mathrm{weierstrassPInverse}\left(4,0,\cos\left(d x+c\right)-i \sin\left(d x+c\right)\right)-2 \left(15 b^{2} \cos\left(d x+c\right)^{3} e^{\frac{3}{2}}+42 a b \cos\left(d x+c\right)^{2} e^{\frac{3}{2}}-42 a b e^{\frac{3}{2}}+5 \left(7 a^{2}-b^{2}\right) \cos\left(d x+c\right) e^{\frac{3}{2}}\right) \sqrt{\sin\left(d x+c\right)}}{105 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/105*(5*sqrt(2)*sqrt(-I)*(7*a^2 + 2*b^2)*e^(3/2)*weierstrassPInverse(4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*sqrt(I)*(7*a^2 + 2*b^2)*e^(3/2)
*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(15*b^2*cos(d
*x + c)^3*e^(3/2) + 42*a*b*cos(d*x + c)^2*e^(3/2) - 42*a*b*e^(3/2) + 5*(7*a
^2 - b^2)*cos(d*x + c)*e^(3/2))*sqrt(sin(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sin(c+dx))^{\frac{3}{2}}\,(a+b\cos(c+dx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(3/2),x)
[Out] Integral((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2*e^(3/2)*sin(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\sin(c+d\,x))^{3/2}\,(a+b\,\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2, x)
```

3.44 $\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx$

Optimal. Leaf size=114

$$\frac{2(5a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}+\frac{14ab(e\sin(c+dx))^{3/2}}{15de}+\frac{2b(a+b\cos(c+dx))(e\sin(c+}{5de}$$

```
[Out] 14/15*a*b*(e*sin(d*x+c))^(3/2)/d/e+2/5*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(3
/2)/d/e-2/5*(5*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1
/2)/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2771, 2748, 2721, 2719}

$$\frac{2(5a^2+2b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}+\frac{14ab(e\sin(c+dx))^{3/2}}{15de}+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*(5*a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(
5*d*Sqrt[Sin[c + d*x]]) + (14*a*b*(e*Sin[c + d*x])^(3/2))/(15*d*e) + (2*b*(
a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(5*d*e)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{5de} + \frac{2}{5}\int \left(\frac{5a^2}{2}+b^2+\frac{7}{2}ab\right.$$

$$= \frac{14ab(e\sin(c+dx))^{3/2}}{15de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{5de}$$

$$= \frac{14ab(e\sin(c+dx))^{3/2}}{15de} + \frac{2b(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{5de}$$

$$= \frac{2(5a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} + \frac{14ab(e\sin}{15}$$

Mathematica [A]
time = 0.30, size = 83, normalized size = 0.73

$$\frac{2\sqrt{e\sin(c+dx)}\left(-3(5a^2+2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)+b(10a+3b\cos(c+dx))\sin^{\frac{3}{2}}(c+dx)\right)}{15d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[e*Sin[c + d*x]]*(-3*(5*a^2 + 2*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4
, 2] + b*(10*a + 3*b*Cos[c + d*x])*Sin[c + d*x]^(3/2)))/(15*d*Sqrt[Sin[c +
d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(130) = 260$.
time = 0.14, size = 294, normalized size = 2.58

method	result
default	$-\frac{e\left(30\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e*(30*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))
*a^2+12*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellip
ticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-15*(-sin(d*x+c)+1)^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/
2))*a^2-6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ell
ipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2+6*b^2*cos(d*x+c)^4+20*a*b*cos
(d*x+c)^3-6*cos(d*x+c)^2*b^2-20*cos(d*x+c)*a*b)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*cos(d*x + c) + a)^2*sqrt(sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 120, normalized size = 1.05

$$\frac{3i\sqrt{2}\sqrt{-i}\,(5a^2+2b^2)e^{\frac{1}{2}}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\sqrt{i}\,(5a^2+2b^2)e^{\frac{1}{2}}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(3b^2\cos(dx+c)e^{\frac{1}{2}}+10abe^{\frac{1}{2}}\right)\sin(dx+c)^{\frac{3}{2}}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*I*sqrt(2)*sqrt(-I)*(5*a^2 + 2*b^2)*e^(1/2)*weierstrassZeta(4, 0, we
ierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*sqrt(
I)*(5*a^2 + 2*b^2)*e^(1/2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
cos(d*x + c) - I*sin(d*x + c))) + 2*(3*b^2*cos(d*x + c)*e^(1/2) + 10*a*b*e^
(1/2))*sin(d*x + c)^(3/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\sin(c+dx)}\,(a+b\cos(c+dx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2*e^(1/2)*sqrt(sin(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\sin\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2, x)
```

3.45 $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=114

$$\frac{2(3a^2+2b^2)\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}}+\frac{10ab\sqrt{e\sin(c+dx)}}{3de}+\frac{2b(a+b\cos(c+dx))\sqrt{e\sin(c+}}{3de}$$

```
[Out] -2/3*(3*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2
*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*si
n(d*x+c))^(1/2)+10/3*a*b*(e*sin(d*x+c))^(1/2)/d/e+2/3*b*(a+b*cos(d*x+c))*(e
*sin(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2771, 2748, 2721, 2720}

$$\frac{2(3a^2+2b^2)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)}{3d\sqrt{e\sin(c+dx)}}+\frac{10ab\sqrt{e\sin(c+dx)}}{3de}+\frac{2b\sqrt{e\sin(c+dx)}\,(a+b\cos(c+dx))}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*(3*a^2 + 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*
d*Sqrt[e*Sin[c + d*x]]) + (10*a*b*Sqrt[e*Sin[c + d*x]])/(3*d*e) + (2*b*(a +
 b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*d*e)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \frac{2b(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de} + \frac{2}{3}\int \frac{\frac{3a^2}{2}+b^2+\frac{5}{2}ab\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{10ab\sqrt{e\sin(c+dx)}}{3de} + \frac{2b(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de} + \frac{1}{3}\left(3a^2+2b^2\right)\int$$
$$= \frac{10ab\sqrt{e\sin(c+dx)}}{3de} + \frac{2b(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3de} + \frac{\left(\left(3a^2+2b^2\right)\sqrt{}\right.}{}$$
$$= \frac{2\left(3a^2+2b^2\right)F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} + \frac{10ab\sqrt{e\sin(c+dx)}}{3de} + \frac{2b(}{}$$

Mathematica [A]
time = 0.42, size = 79, normalized size = 0.69

$$\frac{-2\left(3a^2+2b^2\right)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sqrt{\sin(c+dx)}+2b(6a+b\cos(c+dx))\sin(c+dx)}{3d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
[Out] (-2*(3*a^2 + 2*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]
 + 2*b*(6*a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[e*Sin[c + d*x]])
```

Maple [A]
time = 0.12, size = 170, normalized size = 1.49

method	result
default	$-\frac{3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\operatorname{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)a^2+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2
+2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-2*b^2*cos(d*x+c)^2*sin(d*x+c)-12*a*b
*cos(d*x+c)*sin(d*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*cos(d*x + c) + a)^2/sqrt(sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 105, normalized size = 0.92

$$\frac{\left(\sqrt{2}\,\sqrt{-i}\,(3\,a^2+2\,b^2)\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,\sqrt{i}\,(3\,a^2+2\,b^2)\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c))+2\,(b^2\cos(dx+c)+6\,ab)\sqrt{\sin(dx+c)}\right)e^{\left(-\frac{1}{2}\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*sqrt(-I)*(3*a^2 + 2*b^2)*weierstrassPInverse(4, 0, cos(d*x + c
) + I*sin(d*x + c)) + sqrt(2)*sqrt(I)*(3*a^2 + 2*b^2)*weierstrassPInverse(4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(b^2*cos(d*x + c) + 6*a*b)*sqrt(sin
(d*x + c)))*e^(-1/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2/sqrt(e*sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^2*e^(-1/2)/sqrt(sin(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(1/2),x)
[Out] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(1/2), x)
```

3.46 $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}-\frac{2(a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2ab(e\sin(c+}{de^3}$$

```
[Out] -2*a*b*(e*sin(d*x+c))^(3/2)/d/e^3-2*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))/d/e/(
e*sin(d*x+c))^(1/2)+2*(a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*
x+c))^(1/2)/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2770, 2748, 2721, 2719}

$$-\frac{2(a^2+2b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2ab(e\sin(c+dx))^{3/2}}{de^3}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x]))/(d*e*Sqrt[e*Sin[c + d*x]]) -
 (2*(a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d
*e^2*Sqrt[Sin[c + d*x]]) - (2*a*b*(e*Sin[c + d*x])^(3/2))/(d*e^3)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}} - \frac{2\int\left(\frac{a^2}{2}+b^2+\frac{3}{2}ab\cos(c+dx)\right)\sqrt{e}}{e^2}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}} - \frac{2ab(e\sin(c+dx))^{3/2}}{de^3} - \frac{(a^2+2b^2)}{}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}} - \frac{2ab(e\sin(c+dx))^{3/2}}{de^3} - \frac{\left((a^2+2b^2\right)}{}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{de\sqrt{e\sin(c+dx)}} - \frac{2(a^2+2b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{}}{de^2\sqrt{\sin(c+dx)}}$$

Mathematica [A]
time = 0.26, size = 75, normalized size = 0.64

$$\frac{-4ab-2(a^2+b^2)\cos(c+dx)+2(a^2+2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sqrt{\sin(c+dx)}}{de\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-4*a*b - 2*(a^2 + b^2)*Cos[c + d*x] + 2*(a^2 + 2*b^2)*EllipticE[(-2*c + Pi
 - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]])/(d*e*Sqrt[e*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(140) = 280$.
time = 0.12, size = 283, normalized size = 2.40

method	result

default	$\frac{2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)a^2+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+
4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-2
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-
sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-2*a^2*cos(d*x+c)^2-2*cos(d*x+c)^2*b^2-
4*cos(d*x+c)*a*b)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((b*cos(d*x + c) + a)^2/sin(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 134, normalized size = 1.14

$$\frac{\left(\sqrt{2}\,\sqrt{-i}\,\left(-i\,a^2-2i\,b^2\right)\sin(dx+c)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)))+\sqrt{2}\,\sqrt{i}\,\left(i\,a^2+2i\,b^2\right)\sin(dx+c)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c)))-2\left(2\,ab+\left(a^2+b^2\right)\cos(dx+c)\right)\sqrt{\sin(dx+c)}\right)e^{\left(-\frac{3}{2}\right)}}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*sqrt(-I)*(-I*a^2 - 2*I*b^2)*sin(d*x + c)*weierstrassZeta(4, 0, wei
erstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*sqrt(I)*(I
*a^2 + 2*I*b^2)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 2*(2*a*b + (a^2 + b^2)*cos(d*x + c))*sq
rt(sin(d*x + c)))*e^(-3/2)/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cos(c+dx)\right)^2}{\left(e\sin(c+dx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2/(e*sin(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*e^(-3/2)/sin(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\cos\left(c+d\,x\right))^2}{(e\,\sin\left(c+d\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(3/2), x)
```

3.47 $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}+\frac{2(a^2-2b^2)\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2ab\sqrt{e\sin(c+}}{3de^3}$$

```
[Out] -2/3*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))/d/e/(e*sin(d*x+c))^(3/2)-2/3*(a^2-2*
b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
F(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^2/(e*sin(d*x+c))^
(1/2)-2/3*a*b*(e*sin(d*x+c))^(1/2)/d/e^3
```

Rubi [A]

time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2770, 2748, 2721, 2720}

$$\frac{2(a^2-2b^2)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2ab\sqrt{e\sin(c+dx)}}{3de^3}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x]))/(3*d*e*(e*Sin[c + d*x])^(3/2
)) + (2*(a^2 - 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/
(3*d*e^2*Sqrt[e*Sin[c + d*x]]) - (2*a*b*Sqrt[e*Sin[c + d*x]])/(3*d*e^3)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a^2}{2}+b^2+\frac{1}{2}ab\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}} - \frac{2ab\sqrt{e\sin(c+dx)}}{3de^3} + \frac{(a^2-2b^2)\int$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}} - \frac{2ab\sqrt{e\sin(c+dx)}}{3de^3} + \frac{\left((a^2-2b^2)\right.}{}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{3de(e\sin(c+dx))^{3/2}} + \frac{2(a^2-2b^2)\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{}}{3de^2\sqrt{e\sin(c+dx)}}$$

Mathematica [A]
time = 0.29, size = 76, normalized size = 0.61

$$-\frac{2\left(2ab+(a^2+b^2)\cos(c+dx)+(a^2-2b^2)\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3de(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(2*a*b + (a^2 + b^2)*Cos[c + d*x] + (a^2 - 2*b^2)*EllipticF[(-2*c + Pi
- 2*d*x)/4, 2]*Sin[c + d*x]^(3/2)))/(3*d*e*(e*Sin[c + d*x])^(3/2))
```

Maple [A]
time = 0.13, size = 202, normalized size = 1.63

method	result

default	$-\frac{4ab}{3e(e\sin(dx+c))^{\frac{3}{2}}}-\frac{\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right)\operatorname{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-4/3*a*b/e/(e*sin(d*x+c))^(3/2)-1/3/e^2*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a
^2-2*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*Elli
pticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*a^2*cos(d*x+c)^2*sin(d*x+c)+2*b^
2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((b*cos(d*x + c) + a)^2/sin(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 168, normalized size = 1.35

$$\frac{\sqrt{-i}\left(\sqrt{2}\,(a^2-2\,b^2)\cos(dx+c)^2-\sqrt{2}\,(a^2-2\,b^2)\right)\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{i}\left(\sqrt{2}\,(a^2-2\,b^2)\cos(dx+c)^2-\sqrt{2}\,(a^2-2\,b^2)\right)\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c))+2\,(2\,ab+(a^2+b^2)\cos(dx+c))\sqrt{\sin(dx+c)}}{3\left(d\cos(dx+c)^2\,e^{\frac{5}{2}}-de^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(-I)*(sqrt(2)*(a^2 - 2*b^2)*cos(d*x + c)^2 - sqrt(2)*(a^2 - 2*b^2)
)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(I)*(sqrt(
2)*(a^2 - 2*b^2)*cos(d*x + c)^2 - sqrt(2)*(a^2 - 2*b^2))*weierstrassPInvers
e(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(2*a*b + (a^2 + b^2)*cos(d*x + c
))*sqrt(sin(d*x + c)))/(d*cos(d*x + c)^2*e^(5/2) - d*e^(5/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2/(e*sin(c + d*x))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*e^(-5/2)/sin(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(5/2), x)
```

3.48 $\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}-\frac{2ab}{5de^3\sqrt{e\sin(c+dx)}}-\frac{2(3a^2-2b^2)\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}}-\frac{2(3a^2-2b^2)\,E}{}$$

```
[Out] -2/5*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))/d/e/(e*sin(d*x+c))^(5/2)-2/5*a*b/d/e
^3/(e*sin(d*x+c))^(1/2)-2/5*(3*a^2-2*b^2)*cos(d*x+c)/d/e^3/(e*sin(d*x+c))^(
1/2)+2/5*(3*a^2-2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi
+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)
/d/e^4/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2748, 2716, 2721, 2719}

$$-\frac{2(3a^2-2b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}}-\frac{2(3a^2-2b^2)\cos(c+dx)}{5de^3\sqrt{e\sin(c+dx)}}-\frac{2ab}{5de^3\sqrt{e\sin(c+dx)}}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x]))/(5*d*e*(e*Sin[c + d*x])^(5/2
)) - (2*a*b)/(5*d*e^3*Sqrt[e*Sin[c + d*x]]) - (2*(3*a^2 - 2*b^2)*Cos[c + d*
x])/(5*d*e^3*Sqrt[e*Sin[c + d*x]]) - (2*(3*a^2 - 2*b^2)*EllipticE[(c - Pi/2
 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*e^4*Sqrt[Sin[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+b^2-\frac{1}{2}ab\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{5e^2}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}} - \frac{2ab}{5de^3\sqrt{e\sin(c+dx)}} + \frac{(3a^2-2b^2)}{}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}} - \frac{2ab}{5de^3\sqrt{e\sin(c+dx)}} - \frac{2(3a^2-2b^2}{5de^3\sqrt{e\,s}}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}} - \frac{2ab}{5de^3\sqrt{e\sin(c+dx)}} - \frac{2(3a^2-2b^2}{5de^3\sqrt{e\,s}}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))}{5de(e\sin(c+dx))^{5/2}} - \frac{2ab}{5de^3\sqrt{e\sin(c+dx)}} - \frac{2(3a^2-2b^2}{5de^3\sqrt{e\,s}}$$

Mathematica [A]
time = 0.50, size = 109, normalized size = 0.66

$$-\frac{8ab+(7a^2+2b^2)\cos(c+dx)-3a^2\cos(3(c+dx))+2b^2\cos(3(c+dx))-4(3a^2-2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{5}{2}}(c+dx)}{10de(e\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] -1/10*(8*a*b + (7*a^2 + 2*b^2)*Cos[c + d*x] - 3*a^2*Cos[3*(c + d*x)] + 2*b^
2*Cos[3*(c + d*x)] - 4*(3*a^2 - 2*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, 2]*
Sin[c + d*x]^(5/2))/(d*e*(e*Sin[c + d*x])^(5/2))
```

Maple [A]
time = 0.14, size = 351, normalized size = 2.13

method	result
default	$\frac{-\frac{4ab}{5e(e\sin(dx+c))^{\frac{5}{2}}}+\frac{6\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-4/5*a*b/e/(e*sin(d*x+c))^(5/2)+1/5/e^3*(6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))
*a^2-4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*Ellipt
icE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)
)*a^2+2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*Ellip
ticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2+6*a^2*cos(d*x+c)^4*sin(d*x+c)-4
*b^2*cos(d*x+c)^4*sin(d*x+c)-8*a^2*cos(d*x+c)^2*sin(d*x+c)+2*b^2*cos(d*x+c)
^2*sin(d*x+c))/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((b*cos(d*x + c) + a)^2/sin(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 225, normalized size = 1.36

$$\frac{\sqrt{-i}\left(\sqrt{2}\left(-3i\,a^2+2i\,b^2\right)\cos(dx+c)^2+\sqrt{2}\left(3i\,a^2-2i\,b^2\right)\right)\sin(dx+c)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)))+\sqrt{i}\left(\sqrt{2}\left(3i\,a^2-2i\,b^2\right)\cos(dx+c)^2+\sqrt{2}\left(-3i\,a^2+2i\,b^2\right)\right)\sin(dx+c)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c)))-2\left(\left(3\,a^2-2\,b^2\right)\cos(dx+c)^3-2\,ab-\left(4\,a^2-b^2\right)\cos(dx+c)\right)\sqrt{\sin(dx+c)}}{5\left(d\cos(dx+c)^2\,e^{\frac{7}{2}}-de^{\frac{7}{2}}\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(-I)*(sqrt(2)*(-3*I*a^2 + 2*I*b^2)*cos(d*x + c)^2 + sqrt(2)*(3*I*a
^2 - 2*I*b^2))*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
```

```
 cos(d*x + c) + I*sin(d*x + c))) + sqrt(I)*(sqrt(2)*(3*I*a^2 - 2*I*b^2)*cos
(d*x + c)^2 + sqrt(2)*(-3*I*a^2 + 2*I*b^2))*sin(d*x + c)*weierstrassZeta(4,
 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*((3*a^2 -
 2*b^2)*cos(d*x + c)^3 - 2*a*b - (4*a^2 - b^2)*cos(d*x + c))*sqrt(sin(d*x +
 c)))/((d*cos(d*x + c)^2*e^(7/2) - d*e^(7/2))*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*e^(-7/2)/sin(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\cos\left(c+d\,x\right))^2}{(e\,\sin\left(c+d\,x\right))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^2/(e*sin(c + d*x))^(7/2), x)
```

3.49 $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=242

$$\frac{10a(11a^2+6b^2)\,e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{231d\sqrt{e\sin(c+dx)}}-\frac{10a(11a^2+6b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d}-2$$

```
[Out] -2/77*a*(11*a^2+6*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/d+2/1287*b*(177*a^
2+44*b^2)*(e*sin(d*x+c))^(9/2)/d/e+34/143*a*b*(a+b*cos(d*x+c))*(e*sin(d*x+c
))^(9/2)/d/e+2/13*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(9/2)/d/e-10/231*a*(1
1*a^2+6*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*sin(
d*x+c))^(1/2)-10/231*a*(11*a^2+6*b^2)*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2771, 2941, 2748, 2715, 2721, 2720}

$$\frac{10ae^4(11a^2+6b^2)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)}{231d\sqrt{e\sin(c+dx)}}-\frac{10ae^3(11a^2+6b^2)\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d}+\frac{2b(177a^2+44b^2)\,(e\sin(c+dx))^{9/2}}{1287de}-\frac{2ae(11a^2+6b^2)\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d}+\frac{2b(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))^2}{13de}+\frac{34ab(e\sin(c+dx))^{9/2}(a+b\cos(c+dx))}{143de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] (10*a*(11*a^2 + 6*b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*
x]])/(231*d*Sqrt[e*Sin[c + d*x]]) - (10*a*(11*a^2 + 6*b^2)*e^3*Cos[c + d*x]
*Sqrt[e*Sin[c + d*x]])/(231*d) - (2*a*(11*a^2 + 6*b^2)*e*Cos[c + d*x]*(e*Si
n[c + d*x])^(5/2))/(77*d) + (2*b*(177*a^2 + 44*b^2)*(e*Sin[c + d*x])^(9/2))
/(1287*d*e) + (34*a*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(9/2))/(143*d*e
) + (2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(9/2))/(13*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$
\begin{aligned}
\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}\,dx &= \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{9/2}}{13de} + \frac{2}{13}\int (a+b\cos(c+ \\
&= \frac{34ab(a+b\cos(c+dx))(e\sin(c+dx))^{9/2}}{143de} + \frac{2b(a+b\cos(c+a}{} \\
&= \frac{2b(177a^2+44b^2)\,(e\sin(c+dx))^{9/2}}{1287de} + \frac{34ab(a+b\cos(c+dx))}{143de} \\
&= -\frac{2a(11a^2+6b^2)\,e\cos(c+dx)(e\sin(c+dx))^{5/2}}{77d} + \frac{2b(177a^2+}{} \\
&= -\frac{10a(11a^2+6b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d} - \frac{2a(11a^2+}{} \\
&= -\frac{10a(11a^2+6b^2)\,e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{231d} - \frac{2a(11a^2+}{} \\
&= \frac{10a(11a^2+6b^2)\,e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{231d\sqrt{e\sin(c+dx)}} - \frac{10a}{}
\end{aligned}
$$

Mathematica [A]

time = 2.58, size = 205, normalized size = 0.85

$$
\frac{\left(154b(78a^2+11b^2)\csc^3(c+dx)+\frac{1}{3}(-156a(506a^2+213b^2)\cos(c+dx)-77b(624a^2+73b^2)\cos(2(c+dx))+234a(44a^2-39b^2)\cos(3(c+dx))-154b(-78a^2+b^2)\cos(4(c+dx))+4914ab^2\cos(5(c+dx))+693b^3\cos(6(c+dx)))\csc^3(c+dx)-\frac{2080a(11a^2+6b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)}{\sin^{\frac{7}{2}}(c+dx)}\right)(e\sin(c+dx))^{7/2}}{48048d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] ((154*b*(78*a^2 + 11*b^2)*Csc[c + d*x]^3 + ((-156*a*(506*a^2 + 213*b^2)*Cos
[c + d*x] - 77*b*(624*a^2 + 73*b^2)*Cos[2*(c + d*x)] + 234*a*(44*a^2 - 39*b
^2)*Cos[3*(c + d*x)] - 154*b*(-78*a^2 + b^2)*Cos[4*(c + d*x)] + 4914*a*b^2*
Cos[5*(c + d*x)] + 693*b^3*Cos[6*(c + d*x)])*Csc[c + d*x]^3)/3 - (2080*a*(1
1*a^2 + 6*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2])/Sin[c + d*x]^(7/2))*(e*
Sin[c + d*x])^(7/2))/(48048*d)
```

Maple [A]

time = 0.18, size = 276, normalized size = 1.14

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{9}{2}}\left(9\left(\cos^2(dx+c)\right)b^2+39a^2+4b^2\right)}{117e}-\frac{e^4a\left(-126b^2\left(\cos^6(dx+c)\right)\sin(dx+c)-66a^2\left(\cos^4(dx+c)\right)\sin(dx+c)+216b^2\left(\cos^4(dx+c)\right)\sin(d\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/117/e*b*(e*sin(d*x+c))^(9/2)*(9*cos(d*x+c)^2*b^2+39*a^2+4*b^2)-1/231*e^4
*a*(-126*b^2*cos(d*x+c)^6*sin(d*x+c)-66*a^2*cos(d*x+c)^4*sin(d*x+c)+216*b^2
*cos(d*x+c)^4*sin(d*x+c)+55*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*si
n(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+30*(-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c
)+1)^(1/2),1/2*2^(1/2))*b^2+176*a^2*cos(d*x+c)^2*sin(d*x+c)-30*b^2*cos(d*x+
c)^2*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate((b*cos(d*x + c) + a)^3*sin(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 238, normalized size = 0.98

$$\frac{195\,\sqrt{2}\,\sqrt{-i}\,(11\,a^3+6\,ab^2)e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+195\,\sqrt{2}\,\sqrt{i}\,(11\,a^3+6\,ab^2)e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c))+2\left(693\,b^3\cos(dx+c)^6\,e^{\frac{7}{2}}+2457\,ab^2\cos(dx+c)^5\,e^{\frac{7}{2}}+77\,(39\,a^2b-14\,b^3)\cos(dx+c)^4\,e^{\frac{7}{2}}+117\,(11\,a^3-36\,ab^2)\cos(dx+c)^3\,e^{\frac{7}{2}}-77\,(78\,a^2b-b^3)\cos(dx+c)^2\,e^{\frac{7}{2}}-39\,(88\,a^3-15\,ab^2)\cos(dx+c)\,e^{\frac{7}{2}}+77\,(39\,a^2b+4\,b^3)e^{\frac{7}{2}}\right)\sqrt{\sin(dx+c)}}{9009\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/9009*(195*sqrt(2)*sqrt(-I)*(11*a^3 + 6*a*b^2)*e^(7/2)*weierstrassPInverse
(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 195*sqrt(2)*sqrt(I)*(11*a^3 + 6*a*b
^2)*e^(7/2)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(6
93*b^3*cos(d*x + c)^6*e^(7/2) + 2457*a*b^2*cos(d*x + c)^5*e^(7/2) + 77*(39*
a^2*b - 14*b^3)*cos(d*x + c)^4*e^(7/2) + 117*(11*a^3 - 36*a*b^2)*cos(d*x +
c)^3*e^(7/2) - 77*(78*a^2*b - b^3)*cos(d*x + c)^2*e^(7/2) - 39*(88*a^3 - 15
*a*b^2)*cos(d*x + c)*e^(7/2) + 77*(39*a^2*b + 4*b^3)*e^(7/2))*sqrt(sin(d*x
+ c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*e^(7/2)*sin(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\sin\left(c+d\,x\right))^{7/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3, x)
```

3.50 $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=202

$$\frac{2a(3a^2+2b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2a(3a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{15d}+\frac{2b(43a}{}$$

```
[Out] -2/15*a*(3*a^2+2*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+2/231*b*(43*a^2+1
2*b^2)*(e*sin(d*x+c))^(7/2)/d/e+10/33*a*b*(a+b*cos(d*x+c))*(e*sin(d*x+c))^(
7/2)/d/e+2/11*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(7/2)/d/e-2/5*a*(3*a^2+2*
b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elli
pticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+c)^
(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2771, 2941, 2748, 2715, 2721, 2719}

$$\frac{2ae^2(3a^2+2b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}+\frac{2b(43a^2+12b^2)\,(e\sin(c+dx))^{7/2}}{231de}-\frac{2ae(3a^2+2b^2)\cos(c+dx)(e\sin(c+dx))^{3/2}}{15d}+\frac{2b(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))^2}{11de}+\frac{10ab(e\sin(c+dx))^{7/2}(a+b\cos(c+dx))}{33de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a*(3*a^2 + 2*b^2)*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*
x]])/(5*d*Sqrt[Sin[c + d*x]]) - (2*a*(3*a^2 + 2*b^2)*e*Cos[c + d*x]*(e*Sin[
c + d*x])^(3/2))/(15*d) + (2*b*(43*a^2 + 12*b^2)*(e*Sin[c + d*x])^(7/2))/(2
31*d*e) + (10*a*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2))/(33*d*e) + (
2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2))/(11*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}\,dx = \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}{11de} + \frac{2}{11}\int (a+b\cos(c+$$

$$= \frac{10ab(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}{33de} + \frac{2b(a+b\cos(c+dx}{11$$

$$= \frac{2b(43a^2+12b^2)\,(e\sin(c+dx))^{7/2}}{231de} + \frac{10ab(a+b\cos(c+dx))(e\,s}{33de}$$

$$= -\frac{2a(3a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{15d} + \frac{2b(43a^2+12b}{}$$

$$= -\frac{2a(3a^2+2b^2)\,e\cos(c+dx)(e\sin(c+dx))^{3/2}}{15d} + \frac{2b(43a^2+12b}{}$$

$$= \frac{2a(3a^2+2b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2a(3a}{}$$

Mathematica [A]

time = 1.46, size = 149, normalized size = 0.74

$$-\frac{(e\sin(c+dx))^{5/2}\left(1848(3a^3+2ab^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\,2\right)+(462a(4a^2+b^2)\cos(c+dx)+5b(-396a^2-69b^2+12(33a^2+4b^2)\cos(2(c+dx))+154ab\cos(3(c+dx))+21b^2\cos(4(c+dx))))\sin^{\frac{3}{2}}(c+dx)\right)}{4620d\sin^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/4620*((e*Sin[c + d*x])^(5/2)*(1848*(3*a^3 + 2*a*b^2)*EllipticE[(-2*c + P
i - 2*d*x)/4, 2] + (462*a*(4*a^2 + b^2)*Cos[c + d*x] + 5*b*(-396*a^2 - 69*b
^2 + 12*(33*a^2 + 4*b^2)*Cos[2*(c + d*x)] + 154*a*b*Cos[3*(c + d*x)] + 21*b
^2*Cos[4*(c + d*x)]))*Sin[c + d*x]^(3/2)))/(d*Sin[c + d*x]^(5/2))
```

Maple [A]

time = 0.18, size = 356, normalized size = 1.76

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{7}{2}}\left(7\left(\cos^2(dx+c)\right)b^2+33a^2+4b^2\right)}{77e}-\frac{e^3a\left(10b^2\left(\sin^6(dx+c)\right)+18\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/77/e*b*(e*sin(d*x+c))^(7/2)*(7*cos(d*x+c)^2*b^2+33*a^2+4*b^2)-1/15*e^3*a
*(10*b^2*sin(d*x+c)^6+18*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
```

```
*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+12*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1
)^(1/2),1/2*2^(1/2))*b^2-9*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-6*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))*b^2-6*a^2*sin(d*x+c)^4-14*sin(d*x+c)^4*b^2+6*sin(d*x+
c)^2*a^2+4*sin(d*x+c)^2*b^2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] e^(5/2)*integrate((b*cos(d*x + c) + a)^3*sin(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 194, normalized size = 0.96

$$\frac{231i\sqrt{2}\sqrt{-i}\,(3a^3+2ab^2)e^{\frac{5}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-231i\sqrt{2}\sqrt{i}\,(3a^3+2ab^2)e^{\frac{5}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(105b^3\cos(dx+c)^4e^{\frac{5}{2}}+385ab^2\cos(dx+c)^3e^{\frac{5}{2}}+45(11a^2b-b^3)\cos(dx+c)^2e^{\frac{5}{2}}+231(a^3-ab^2)\cos(dx+c)e^{\frac{5}{2}}-15(33a^2b+4b^3)e^{\frac{5}{2}}\right)\sin(dx+c)^{\frac{3}{2}}}{1155d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 1/1155*(231*I*sqrt(2)*sqrt(-I)*(3*a^3 + 2*a*b^2)*e^(5/2)*weierstrassZeta(4,
 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 231*I*sqrt(
2)*sqrt(I)*(3*a^3 + 2*a*b^2)*e^(5/2)*weierstrassZeta(4, 0, weierstrassPInve
rse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(105*b^3*cos(d*x + c)^4*e^(5/
2) + 385*a*b^2*cos(d*x + c)^3*e^(5/2) + 45*(11*a^2*b - b^3)*cos(d*x + c)^2*
e^(5/2) + 231*(a^3 - a*b^2)*cos(d*x + c)*e^(5/2) - 15*(33*a^2*b + 4*b^3)*e^
(5/2))*sin(d*x + c)^(3/2))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*e^(5/2)*sin(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3, x)
```

3.51 $\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=202

$$\frac{2a(7a^2+6b^2)\,e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}}-\frac{2a(7a^2+6b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d}+\frac{2b(89a^2}{}$$

```
[Out] 2/315*b*(89*a^2+28*b^2)*(e*sin(d*x+c))^(5/2)/d/e+26/63*a*b*(a+b*cos(d*x+c))
*(e*sin(d*x+c))^(5/2)/d/e+2/9*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(5/2)/d/e
-2/21*a*(7*a^2+6*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/
d/(e*sin(d*x+c))^(1/2)-2/21*a*(7*a^2+6*b^2)*e*cos(d*x+c)*(e*sin(d*x+c))^(1/
2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2771, 2941, 2748, 2715, 2721, 2720}

$$\frac{2ae^2(7a^2+6b^2)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21d\sqrt{e\sin(c+dx)}}+\frac{2b(89a^2+28b^2)\,(e\sin(c+dx))^{5/2}}{315de}-\frac{2ae(7a^2+6b^2)\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d}+\frac{2b(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}{9de}+\frac{26ab(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}{63de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*a*(7*a^2 + 6*b^2)*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/(21*d*Sqrt[e*Sin[c + d*x]]) - (2*a*(7*a^2 + 6*b^2)*e*Cos[c + d*x]*Sqrt[e
*Sin[c + d*x]])/(21*d) + (2*b*(89*a^2 + 28*b^2)*(e*Sin[c + d*x])^(5/2))/(31
5*d*e) + (26*a*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2))/(63*d*e) + (2
*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2))/(9*d*e)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}\,dx &= \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}{9de}+\frac{2}{9}\int (a+b\cos(c+\\
&= \frac{26ab(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}{63de}+\frac{2b(a+b\cos(c+a}{}\\
&= \frac{2b(89a^2+28b^2)\,(e\sin(c+dx))^{5/2}}{315de}+\frac{26ab(a+b\cos(c+dx))(}{63de}\\
&= -\frac{2a(7a^2+6b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d}+\frac{2b(89a^2+28}{}\\
&= -\frac{2a(7a^2+6b^2)\,e\cos(c+dx)\sqrt{e\sin(c+dx)}}{21d}+\frac{2b(89a^2+28}{}\\
&= \frac{2a(7a^2+6b^2)\,e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21d\sqrt{e\sin(c+dx)}}-\frac{2a(7a}{}\end{aligned}$$

Mathematica [A]
time = 1.30, size = 147, normalized size = 0.73

$$\frac{\left(-20a(28a^2+15b^2)\cot(c+dx)-\frac{2}{3}b(-756a^2-147b^2+28(27a^2+4b^2)\cos(2(c+dx))+270ab\cos(3(c+dx))+35b^2\cos(4(c+dx)))\csc(c+dx)-\frac{80a(7a^2+6b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)}{\sin^{\frac{3}{2}}(c+dx)}\right)(e\sin(c+dx))^{3/2}}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((-20*a*(28*a^2 + 15*b^2)*Cot[c + d*x] - (2*b*(-756*a^2 - 147*b^2 + 28*(27*
a^2 + 4*b^2)*Cos[2*(c + d*x)] + 270*a*b*Cos[3*(c + d*x)] + 35*b^2*Cos[4*(c
+ d*x)])*Csc[c + d*x])/3 - (80*a*(7*a^2 + 6*b^2)*EllipticF[(-2*c + Pi - 2*d
*x)/4, 2])/Sin[c + d*x]^(3/2))*(e*Sin[c + d*x])^(3/2))/(840*d)
```

Maple [A]
time = 0.17, size = 291, normalized size = 1.44

method	result
default	$-\frac{e^2\left(70b^3\left(\cos^5(dx+c)\right)\sin(dx+c)+270a\,b^2\left(\cos^4(dx+c)\right)\sin(dx+c)+105\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^2*(70*b^3*cos(d*x+c)^5*sin(d*x+c)+
270*a*b^2*cos(d*x+c)^4*sin(d*x+c)+105*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
```

```
)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^3+9
0*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a*b^2+378*a^2*b*cos(d*x+c)^3*sin(d*x+c)-1
4*b^3*cos(d*x+c)^3*sin(d*x+c)+210*a^3*cos(d*x+c)^2*sin(d*x+c)-90*a*b^2*cos(
d*x+c)^2*sin(d*x+c)-378*a^2*b*cos(d*x+c)*sin(d*x+c)-56*b^3*cos(d*x+c)*sin(d
*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((b*cos(d*x + c) + a)^3*sin(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 190, normalized size = 0.94

$$\frac{15\sqrt{2}\,\sqrt{-i}\,(7\,a^3+6\,ab^2)e^{\frac{3}{2}}\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+15\sqrt{2}\,\sqrt{i}\,(7\,a^3+6\,ab^2)e^{\frac{3}{2}}\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c))-2\left(35\,b^3\cos(dx+c)^4\,e^{\frac{3}{2}}+135\,ab^2\cos(dx+c)^3\,e^{\frac{3}{2}}+7\,(27\,a^2b-b^3)\cos(dx+c)^2\,e^{\frac{3}{2}}+15\,(7\,a^3-3\,ab^2)\cos(dx+c)\,e^{\frac{3}{2}}-7\,(27\,a^2b+4\,b^3)e^{\frac{3}{2}}\right)\sqrt{\sin(dx+c)}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(15*sqrt(2)*sqrt(-I)*(7*a^3 + 6*a*b^2)*e^(3/2)*weierstrassPInverse(4,
 0, cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*sqrt(I)*(7*a^3 + 6*a*b^2)*e
^(3/2)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(35*b^3
*cos(d*x + c)^4*e^(3/2) + 135*a*b^2*cos(d*x + c)^3*e^(3/2) + 7*(27*a^2*b -
b^3)*cos(d*x + c)^2*e^(3/2) + 15*(7*a^3 - 3*a*b^2)*cos(d*x + c)*e^(3/2) - 7
*(27*a^2*b + 4*b^3)*e^(3/2))*sqrt(sin(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sin(c+dx))^{\frac{3}{2}}\,(a+b\cos(c+dx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3*e^(3/2)*sin(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\sin\left(c+d\,x\right))^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3, x)
```

3.52 $\int (a+b\cos (c+dx))^3 \sqrt{e\sin (c+dx)}\, dx$

Optimal. Leaf size=161

$$\frac{2a(5a^2+6b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}+\frac{2b(57a^2+20b^2)\,(e\sin(c+dx))^{3/2}}{105de}+\frac{22ab(a+b\cos(c+}{3}$$

```
[Out] 2/105*b*(57*a^2+20*b^2)*(e*sin(d*x+c))^(3/2)/d/e+22/35*a*b*(a+b*cos(d*x+c))
*(e*sin(d*x+c))^(3/2)/d/e+2/7*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^(3/2)/d/e
-2/5*a*(5*a^2+6*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d
/sin(d*x+c)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2771, 2941, 2748, 2721, 2719}

$$\frac{2b(57a^2+20b^2)\,(e\sin(c+dx))^{3/2}}{105de}+\frac{2a(5a^2+6b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}+\frac{2b(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}{7de}+\frac{22ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{35de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*a*(5*a^2 + 6*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])
/(5*d*Sqrt[Sin[c + d*x]]) + (2*b*(57*a^2 + 20*b^2)*(e*Sin[c + d*x])^(3/2))/
(105*d*e) + (22*a*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(35*d*e) +
 (2*b*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2))/(7*d*e)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}\,dx = \frac{2b(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}{7de}+\frac{2}{7}\int (a+b\cos(c+d$$

$$=\frac{22ab(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}{35de}+\frac{2b(a+b\cos(c+d}{7}$$

$$=\frac{2b(57a^2+20b^2)(e\sin(c+dx))^{3/2}}{105de}+\frac{22ab(a+b\cos(c+dx))(e}{35de}$$

$$=\frac{2b(57a^2+20b^2)(e\sin(c+dx))^{3/2}}{105de}+\frac{22ab(a+b\cos(c+dx))(e}{35de}$$

$$=\frac{2a(5a^2+6b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}+\frac{2b(57a^2}{}$$

Mathematica [A]
time = 0.59, size = 105, normalized size = 0.65

$$\frac{\sqrt{e\sin(c+dx)}\left(-42(5a^3+6ab^2)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+b(210a^2+55b^2+126ab\cos(c+dx)+15b^2\cos(2(c+dx)))\sin^{\frac{3}{2}}(c+dx)\right)}{105d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[e*Sin[c + d*x]]*(-42*(5*a^3 + 6*a*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/
4, 2] + b*(210*a^2 + 55*b^2 + 126*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*x)
])*Sin[c + d*x]^(3/2)))/(105*d*Sqrt[Sin[c + d*x]])
```

Maple [A]
time = 0.16, size = 315, normalized size = 1.96

method	result
default	$\frac{\frac{2b(e\sin(dx+c))^{\frac{3}{2}}\left(3\left(\cos^2(dx+c)\right)b^2+21a^2+4b^2\right)}{21e}-\frac{ae\left(10\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}(dx+c)\right)\text{Ellipti}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/21*b/e*(e*sin(d*x+c))^(3/2)*(3*cos(d*x+c)^2*b^2+21*a^2+4*b^2)-1/5*a*e*(1
0*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+12*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^
2-5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF
((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b
^2+6*sin(d*x+c)^4*b^2-6*sin(d*x+c)^2*b^2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((b*cos(d*x + c) + a)^3*sqrt(sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 148, normalized size = 0.92

$$\frac{21i\sqrt{2}\sqrt{-i}\left(5a^3+6ab^2\right)e^{\frac{1}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}\sqrt{i}\left(5a^3+6ab^2\right)e^{\frac{1}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(15b^3\cos(dx+c)^2e^{\frac{1}{2}}+63ab^2\cos(dx+c)e^{\frac{1}{2}}+5\left(21a^2b+4b^3\right)e^{\frac{1}{2}}\right)\sin(dx+c)^{\frac{3}{2}}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(21*I*sqrt(2)*sqrt(-I)*(5*a^3 + 6*a*b^2)*e^(1/2)*weierstrassZeta(4, 0
, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*
```

```
sqrt(I)*(5*a^3 + 6*a*b^2)*e^(1/2)*weierstrassZeta(4, 0, weierstrassPInverse
(4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*b^3*cos(d*x + c)^2*e^(1/2) +
 63*a*b^2*cos(d*x + c)*e^(1/2) + 5*(21*a^2*b + 4*b^3)*e^(1/2))*sin(d*x + c)
^(3/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\sin\left(c+dx\right)}\left(a+b\cos\left(c+dx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(e*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3*e^(1/2)*sqrt(sin(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\sin\left(c+d\,x\right)}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3, x)
```

3.53 $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=157

$$\frac{2a(a^2+2b^2)\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}+\frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{5de}+\frac{6ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{5de}$$

```
[Out] -2*a*(a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/(e*sin(
d*x+c))^(1/2)+2/5*b*(11*a^2+4*b^2)*(e*sin(d*x+c))^(1/2)/d/e+6/5*a*b*(a+b*co
s(d*x+c))*(e*sin(d*x+c))^(1/2)/d/e+2/5*b*(a+b*cos(d*x+c))^2*(e*sin(d*x+c))^
(1/2)/d/e
```

Rubi [A]
time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2771, 2941, 2748, 2721, 2720}

$$\frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{5de}+\frac{2a(a^2+2b^2)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{e\sin(c+dx)}}+\frac{2b\sqrt{e\sin(c+dx)}\,(a+b\cos(c+dx))^2}{5de}+\frac{6ab\sqrt{e\sin(c+dx)}\,(a+b\cos(c+dx))}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*a*(a^2 + 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*
Sqrt[e*Sin[c + d*x]]) + (2*b*(11*a^2 + 4*b^2)*Sqrt[e*Sin[c + d*x]])/(5*d*e)
 + (6*a*b*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(5*d*e) + (2*b*(a + b*
Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]])/(5*d*e)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
```

```
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\cos(c+dx))^3}{\sqrt{e\sin(c+dx)}}\,dx &= \frac{2b(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}{5de}+\frac{2}{5}\int \frac{(a+b\cos(c+dx))\left(\frac{5a^2}{2}+2b^2+\right.}{\sqrt{e\sin(c+dx)}}\\
&= \frac{6ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{5de}+\frac{2b(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx}}{5de}\\
&= \frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{5de}+\frac{6ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{5de}+\frac{2l}{}\\
&= \frac{2b(11a^2+4b^2)\sqrt{e\sin(c+dx)}}{5de}+\frac{6ab(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}{5de}+\frac{2l}{}\\
&= \frac{2a(a^2+2b^2)F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}+\frac{2b(11a^2+4b^2)\sqrt{e\sin(c+}}{5de}
\end{aligned}
$$

Mathematica [A]

time = 0.77, size = 98, normalized size = 0.62

$$
\frac{-10a(a^2+2b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sqrt{\sin(c+dx)}+b(30a^2+9b^2+10ab\cos(c+dx)+b^2\cos(2(c+dx)))\sin(c+dx)}{5d\sqrt{e\sin(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (-10*a*(a^2 + 2*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]
 + b*(30*a^2 + 9*b^2 + 10*a*b*Cos[c + d*x] + b^2*Cos[2*(c + d*x)])*Sin[c +
d*x])/(5*d*Sqrt[e*Sin[c + d*x]])
```

Maple [A]
time = 0.13, size = 210, normalized size = 1.34

method	result
default	$-\frac{5\sqrt{-\sin\left(dx+c\right)+1}\,\sqrt{2\sin\left(dx+c\right)+2}\,\left(\sqrt{\sin}\left(dx+c\right)\right)\mathrm{EllipticF}\left(\sqrt{-\sin\left(dx+c\right)+1},\frac{\sqrt{2}}{2}\right)a^{3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^3
+10*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF
((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a*b^2-2*b^3*cos(d*x+c)^3*sin(d*x+c)-10*
a*b^2*cos(d*x+c)^2*sin(d*x+c)-30*a^2*b*cos(d*x+c)*sin(d*x+c)-8*b^3*cos(d*x+
c)*sin(d*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((b*cos(d*x + c) + a)^3/sqrt(sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 126, normalized size = 0.80

$$\frac{\left(5\sqrt{2}\,\sqrt{-i}\,\left(a^{3}+2\,ab^{2}\right)\mathrm{weierstrassPInverse}\left(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\,\sqrt{i}\,\left(a^{3}+2\,ab^{2}\right)\mathrm{weierstrassPInverse}\left(4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+2\left(b^{3}\cos\left(dx+c\right)^{2}+5\,ab^{2}\cos\left(dx+c\right)+15\,a^{2}b+4\,b^{3}\right)\sqrt{\sin\left(dx+c\right)}\right)e^{\left(-\frac{1}{2}\right)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(5*sqrt(2)*sqrt(-I)*(a^3 + 2*a*b^2)*weierstrassPInverse(4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + 5*sqrt(2)*sqrt(I)*(a^3 + 2*a*b^2)*weierstrassPInver
```

```
se(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(b^3*cos(d*x + c)^2 + 5*a*b^2*c
os(d*x + c) + 15*a^2*b + 4*b^3)*sqrt(sin(d*x + c)))*e^(-1/2)/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\cos(c + dx))^3}{\sqrt{e\sin(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(1/2),x)
[Out] Integral((a + b*cos(c + d*x))**3/sqrt(e*sin(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3*e^(-1/2)/sqrt(sin(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\cos(c + d\,x))^3}{\sqrt{e\,\sin(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(1/2),x)
[Out] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(1/2), x)
```

3.54 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}-\frac{2a(a^2+6b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2b(3a^2+4b}{}$$

```
[Out] -2/3*b*(3*a^2+4*b^2)*(e*sin(d*x+c))^(3/2)/d/e^3-2*a*b*(a+b*cos(d*x+c))*(e*s
in(d*x+c))^(3/2)/d/e^3-2*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x
+c))^(1/2)+2*a*(a^2+6*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/
4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(
1/2)/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2941, 2748, 2721, 2719}

$$-\frac{2b(3a^2+4b^2)(e\sin(c+dx))^{3/2}}{3de^3}-\frac{2a(a^2+6b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2ab(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}{de^3}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(d*e*Sqrt[e*Sin[c + d*x]])
 - (2*a*(a^2 + 6*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]]
)/(d*e^2*Sqrt[Sin[c + d*x]]) - (2*b*(3*a^2 + 4*b^2)*(e*Sin[c + d*x])^(3/2))
/(3*d*e^3) - (2*a*b*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(d*e^3)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&& !LtQ[p, -1] && IntegerQ[2*m] && !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{3/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}} - \frac{2\int (a+b\cos(c+dx))\left(\frac{a^2}{2}+2b^2\right.}{}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}} - \frac{2ab(a+b\cos(c+dx))(e\sin(c+}{de^3}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}} - \frac{2b(3a^2+4b^2)(e\sin(c+dx))^{3/2}}{3de^3}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}} - \frac{2b(3a^2+4b^2)(e\sin(c+dx))^{3/2}}{3de^3}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a(a^2+6b^2)E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right.}{de^2\sqrt{\sin(c+dx}}$$

Mathematica [A]
time = 0.37, size = 101, normalized size = 0.61

$$-\frac{2\left(9a^2b+3b^3+3a(a^2+3b^2)\cos(c+dx)-3a(a^2+6b^2)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sqrt{\sin(c+dx)}+b^3\sin^2(c+dx)\right)}{3de\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(9*a^2*b + 3*b^3 + 3*a*(a^2 + 3*b^2)*Cos[c + d*x] - 3*a*(a^2 + 6*b^2)*E
llipticE[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]] + b^3*Sin[c + d*x]^2)
)/(3*d*e*Sqrt[e*Sin[c + d*x]])
```

Maple [A]
time = 0.14, size = 313, normalized size = 1.90

method	result
default	$\frac{6\sqrt{-\sin\left(dx+c\right)+1}\,\sqrt{2\sin\left(dx+c\right)+2}\,\left(\sqrt{\sin}\left(dx+c\right)\right)\mathrm{EllipticE}\left(\sqrt{-\sin\left(dx+c\right)+1},\frac{\sqrt{2}}{2}\right)a^{3}+36}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/e/(e*sin(d*x+c))^(1/2)/cos(d*x+c)*(6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^
3+36*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elliptic
E((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a*b^2-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)
)*a^3-18*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elli
pticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a*b^2+2*b^3*cos(d*x+c)^3-6*a^3*cos
(d*x+c)^2-18*b^2*a*cos(d*x+c)^2-18*a^2*b*cos(d*x+c)-8*b^3*cos(d*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((b*cos(d*x + c) + a)^3/sin(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 162, normalized size = 0.98

$$-\frac{\left(3\sqrt{2}\sqrt{-i}\left(i\,a^{3}+6i\,ab^{2}\right)\sin\left(dx+c\right)\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\sqrt{i}\left(-i\,a^{3}-6i\,ab^{2}\right)\sin\left(dx+c\right)\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-2\left(b^{3}\cos\left(dx+c\right)^{2}-9\,a^{2}b-4\,b^{3}-3\left(a^{3}+3\,ab^{2}\right)\cos\left(dx+c\right)\right)\sqrt{\sin\left(dx+c\right)}\right)e^{\left(-\frac{3}{2}\right)}}{3\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*sqrt(2)*sqrt(-I)*(I*a^3 + 6*I*a*b^2)*sin(d*x + c)*weierstrassZeta(4
, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*
sqrt(I)*(-I*a^3 - 6*I*a*b^2)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrass
PInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(b^3*cos(d*x + c)^2 - 9*
a^2*b - 4*b^3 - 3*(a^3 + 3*a*b^2)*cos(d*x + c))*sqrt(sin(d*x + c)))*e^(-3/2
)/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\cos(c + dx))^3}{(e\sin(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**3/(e*sin(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*e^(-3/2)/sin(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\cos(c + d\,x))^3}{(e\,\sin(c + d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(3/2), x)
```

3.55 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}+\frac{2a(a^2-6b^2)\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2b(a^2+4b^2)}{3}$$

```
[Out] -2/3*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*(a^
2-6*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elli
pticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^2/(e*sin(d*x+
c))^(1/2)-2/3*b*(a^2+4*b^2)*(e*sin(d*x+c))^(1/2)/d/e^3-2/3*a*b*(a+b*cos(d*x
+c))*(e*sin(d*x+c))^(1/2)/d/e^3
```

Rubi [A]
time = 0.16, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2941, 2748, 2721, 2720}

$$-\frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{3de^3}+\frac{2a(a^2-6b^2)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\,2\right)}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2ab\sqrt{e\sin(c+dx)}\,(a+b\cos(c+dx))}{3de^3}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(3*d*e*(e*Sin[c + d*x])^(3
/2)) + (2*a*(a^2 - 6*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x
]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]]) - (2*b*(a^2 + 4*b^2)*Sqrt[e*Sin[c + d*x]
])/(3*d*e^3) - (2*a*b*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*d*e^3)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{5/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2\int \frac{(a+b\cos(c+dx))\left(-\frac{a^2}{2}+2b^2+\frac{3}{2}ab\cos(c\cdots\right.}{\sqrt{e\sin(c+dx)}}}{3e^2}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2ab(a+b\cos(c+dx))\sqrt{e\sin(c+\cdots}}{3de^3}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{3de^3} - \cdots$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2b(a^2+4b^2)\sqrt{e\sin(c+dx)}}{3de^3} - \cdots$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{3de(e\sin(c+dx))^{3/2}} + \frac{2a(a^2-6b^2)F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\cdots\right.}{3de^2\sqrt{e\sin(c+dx\cdots}}$$

Mathematica [A]
time = 0.92, size = 102, normalized size = 0.60

$$-\frac{6a^2b+5b^3+2a(a^2+3b^2)\cos(c+dx)-3b^3\cos(2(c+dx))+2a(a^2-6b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{3}{2}}(c+dx)}{3de(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/3*(6*a^2*b + 5*b^3 + 2*a*(a^2 + 3*b^2)*Cos[c + d*x] - 3*b^3*Cos[2*(c + d
*x)] + 2*a*(a^2 - 6*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(
3/2))/(d*e*(e*Sin[c + d*x])^(3/2))
```

Maple [A]
time = 0.14, size = 226, normalized size = 1.34

method	result
default	$-\frac{2b\left(-3\left(\cos^{2}(dx+c)\right)b^{2}+3a^{2}+4b^{2}\right)}{3e(e\sin(dx+c))^{\frac{3}{2}}}-\frac{a\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(\cdots\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/3*b/e/(e*sin(d*x+c))^(3/2)*(-3*cos(d*x+c)^2*b^2+3*a^2+4*b^2)-1/3*a/e^2*
((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-
sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-6*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+
2*a^2*cos(d*x+c)^2*sin(d*x+c)+6*b^2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^2/c
os(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((b*cos(d*x + c) + a)^3/sin(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 196, normalized size = 1.16

$$\frac{\sqrt{-i}\left(\sqrt{2}\left(a^{3}-6\,ab^{2}\right)\cos\left(dx+c\right)^{2}-\sqrt{2}\left(a^{3}-6\,ab^{2}\right)\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{i}\left(\sqrt{2}\left(a^{3}-6\,ab^{2}\right)\cos\left(dx+c\right)^{2}-\sqrt{2}\left(a^{3}-6\,ab^{2}\right)\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-2\left(3\,b^{3}\cos\left(dx+c\right)^{2}-3\,a^{2}b-4\,b^{3}-\left(a^{3}+3\,ab^{2}\right)\cos\left(dx+c\right)\right)\sqrt{\sin\left(dx+c\right)}}{3\left(d\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}-de^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(-I)*(sqrt(2)*(a^3 - 6*a*b^2)*cos(d*x + c)^2 - sqrt(2)*(a^3 - 6*a*
b^2))*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(I)*(s
```

```
qrt(2)*(a^3 - 6*a*b^2)*cos(d*x + c)^2 - sqrt(2)*(a^3 - 6*a*b^2))*weierstras
sPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(3*b^3*cos(d*x + c)^2 -
3*a^2*b - 4*b^3 - (a^3 + 3*a*b^2)*cos(d*x + c))*sqrt(sin(d*x + c)))/(d*cos(
d*x + c)^2*e^(5/2) - d*e^(5/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\cos{(c + dx)})^3}{(e\sin{(c + dx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(5/2),x)
[Out] Integral((a + b*cos(c + d*x))**3/(e*sin(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c) + a)^3*e^(-5/2)/sin(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(5/2),x)
[Out] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(5/2), x)
```

3.56 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=192

$$-\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}+\frac{2(a+b\cos(c+dx))\left(ab-(3a^2-4b^2)\cos(c+dx)\right)}{5de^3\sqrt{e\sin(c+dx)}}-\frac{6a(a^2-2b^2}{}$$

[Out] -2/5*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x+c))^(5/2)-2/5*b*(3*a^2-4*b^2)*(e*sin(d*x+c))^(3/2)/d/e^5+2/5*(a+b*cos(d*x+c))*(a*b-(3*a^2-4*b^2)*cos(d*x+c))/d/e^3/(e*sin(d*x+c))^(1/2)+6/5*a*(a^2-2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/e^4/sin(d*x+c)^(1/2)

Rubi [A]
time = 0.18, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2940, 2748, 2721, 2719}

$$-\frac{2b(3a^2-4b^2)(e\sin(c+dx))^{3/2}}{5de^5}-\frac{6a(a^2-2b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\sqrt{\sin(c+dx)}}+\frac{2\left(ab-(3a^2-4b^2)\cos(c+dx)\right)(a+b\cos(c+dx))}{5de^3\sqrt{e\sin(c+dx)}}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(7/2),x]

[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(5*d*e*(e*Sin[c + d*x])^(5/2)) + (2*(a + b*Cos[c + d*x])*(a*b - (3*a^2 - 4*b^2)*Cos[c + d*x]))/(5*d*e^3*Sqrt[e*Sin[c + d*x]]) - (6*a*(a^2 - 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*e^4*Sqrt[Sin[c + d*x]]) - (2*b*(3*a^2 - 4*b^2)*(e*Sin[c + d*x])^(3/2))/(5*d*e^5)

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
```

```
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{7/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}} - \frac{2\int \frac{(a+b\cos(c+dx))\left(-\frac{3a^2}{2}+2b^2+\frac{1}{2}ab\cos(c\cdots\right.}{(e\sin(c+dx))^{3/2}}}{5e^2}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}} + \frac{2(a+b\cos(c+dx))\left(ab-(3a^2-\cdots\right.}{5de^3\sqrt{e\sin(c+d\cdots}}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}} + \frac{2(a+b\cos(c+dx))\left(ab-(3a^2-\cdots\right.}{5de^3\sqrt{e\sin(c+d\cdots}}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}} + \frac{2(a+b\cos(c+dx))\left(ab-(3a^2-\cdots\right.}{5de^3\sqrt{e\sin(c+d\cdots}}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{5de(e\sin(c+dx))^{5/2}} + \frac{2(a+b\cos(c+dx))\left(ab-(3a^2-\cdots\right.}{5de^3\sqrt{e\sin(c+d\cdots}}$$

Mathematica [A]
time = 0.67, size = 130, normalized size = 0.68

$$-\frac{12a^2b-6b^3+a(7a^2+6b^2)\cos(c+dx)+10b^3\cos(2(c+dx))-3a^3\cos(3(c+dx))+6ab^2\cos(3(c+dx))-12a(a^2-2b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{5}{2}}(c+dx)}{10de(e\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(7/2),x]
```

```
[Out] -1/10*(12*a^2*b - 6*b^3 + a*(7*a^2 + 6*b^2)*Cos[c + d*x] + 10*b^3*Cos[2*(c
+ d*x)] - 3*a^3*Cos[3*(c + d*x)] + 6*a*b^2*Cos[3*(c + d*x)] - 12*a*(a^2 - 2
*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(5/2))/(d*e*(e*Sin[c
 + d*x])^(5/2))
```

Maple [A]
time = 0.15, size = 375, normalized size = 1.95

method	result
default	$-\frac{2b\left(5\left(\cos^{2}(dx+c)\right)b^{2}+3a^{2}-4b^{2}\right)}{5e(e\sin(dx+c))^{\frac{5}{2}}}+\frac{a\left(6\sqrt{-\sin\left(dx+c\right)+1}\,\sqrt{2\sin\left(dx+c\right)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)\mathrm{EllipticE}\left(\sqrt{-\sin\left(d\right.}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/5*b/e/(e*sin(d*x+c))^(5/2)*(5*cos(d*x+c)^2*b^2+3*a^2-4*b^2)+1/5*a/e^3*(
6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-12*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^
2-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF
((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b
^2+6*a^2*cos(d*x+c)^4*sin(d*x+c)-12*b^2*cos(d*x+c)^4*sin(d*x+c)-8*a^2*cos(d
*x+c)^2*sin(d*x+c)+6*b^2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^3/cos(d*x+c)/(
e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate((b*cos(d*x + c) + a)^3/sin(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 251, normalized size = 1.31

$$-\frac{3\sqrt{-i}\left(\sqrt{2}\,(i\,a^{3}-2i\,ab^{2})\cos\left(dx+c\right)^{2}+\sqrt{2}\,(-i\,a^{3}+2i\,ab^{2})\right)\sin\left(dx+c\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+3\sqrt{i}\left(\sqrt{2}\,(-i\,a^{3}+2i\,ab^{2})\cos\left(dx+c\right)^{2}+\sqrt{2}\,(i\,a^{3}-2i\,ab^{2})\right)\sin\left(dx+c\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-2\left(5\,b^{3}\cos\left(dx+c\right)^{2}-3\,(a^{3}-2\,ab^{2})\cos\left(dx+c\right)^{3}+3\,a^{2}b-4\,b^{3}+(4\,a^{3}-3\,ab^{2})\cos\left(dx+c\right)\right)\sqrt{\sin\left(dx+c\right)}}{5\left(d\cos\left(dx+c\right)^{2}e^{\frac{7}{2}}-de^{\frac{7}{2}}\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*sqrt(-I)*(sqrt(2)*(I*a^3 - 2*I*a*b^2)*cos(d*x + c)^2 + sqrt(2)*(-I*
a^3 + 2*I*a*b^2))*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(I)*(sqrt(2)*(-I*a^3 + 2*I*a*b^
2)*cos(d*x + c)^2 + sqrt(2)*(I*a^3 - 2*I*a*b^2))*sin(d*x + c)*weierstrassZe
ta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(5*b
^3*cos(d*x + c)^2 - 3*(a^3 - 2*a*b^2)*cos(d*x + c)^3 + 3*a^2*b - 4*b^3 + (4
*a^3 - 3*a*b^2)*cos(d*x + c))*sqrt(sin(d*x + c)))/((d*cos(d*x + c)^2*e^(7/2
) - d*e^(7/2))*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*e^(-7/2)/sin(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(7/2), x)
```

3.57 $\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}-\frac{2(a+b\cos(c+dx))\left(ab+(5a^2-4b^2)\cos(c+dx)\right)}{21de^3(e\sin(c+dx))^{3/2}}+\frac{2a(5a^2-6b}{}$$

[Out] -2/7*(b+a*cos(d*x+c))*(a+b*cos(d*x+c))^2/d/e/(e*sin(d*x+c))^(7/2)-2/21*(a+b*cos(d*x+c))*(a*b+(5*a^2-4*b^2)*cos(d*x+c))/d/e^3/(e*sin(d*x+c))^(3/2)-2/21*a*(5*a^2-6*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^4/(e*sin(d*x+c))^(1/2)-2/21*b*(5*a^2-4*b^2)*(e*sin(d*x+c))^(1/2)/d/e^5

Rubi [A]

time = 0.18, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2770, 2940, 2748, 2721, 2720}

$$-\frac{2b(5a^2-4b^2)\sqrt{e\sin(c+dx)}}{21de^5}+\frac{2a(5a^2-6b^2)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21de^4\sqrt{e\sin(c+dx)}}-\frac{2\left((5a^2-4b^2)\cos(c+dx)+ab\right)(a+b\cos(c+dx))}{21de^3(e\sin(c+dx))^{3/2}}-\frac{2(a\cos(c+dx)+b)(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(9/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a + b*Cos[c + d*x])^2)/(7*d*e*(e*Sin[c + d*x])^(7/2)) - (2*(a + b*Cos[c + d*x])*(a*b + (5*a^2 - 4*b^2)*Cos[c + d*x]))/(21*d*e^3*(e*Sin[c + d*x])^(3/2)) + (2*a*(5*a^2 - 6*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*d*e^4*Sqrt[e*Sin[c + d*x]]) - (2*b*(5*a^2 - 4*b^2)*Sqrt[e*Sin[c + d*x]])/(21*d*e^5)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
```

```
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{(e\sin(c+dx))^{9/2}}\,dx = -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2\int \frac{(a+b\cos(c+dx))\left(-\frac{5a^2}{2}+2b^2-\frac{1}{2}ab\cos(c}{(e\sin(c+dx))^{5/2}}}{7e^2}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2(a+b\cos(c+dx))\,(ab+(5a^2-}{21de^3(e\sin(c+dx)}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2(a+b\cos(c+dx))\,(ab+(5a^2-}{21de^3(e\sin(c+dx)}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2(a+b\cos(c+dx))\,(ab+(5a^2-}{21de^3(e\sin(c+dx)}$$

$$= -\frac{2(b+a\cos(c+dx))(a+b\cos(c+dx))^2}{7de(e\sin(c+dx))^{7/2}} - \frac{2(a+b\cos(c+dx))\,(ab+(5a^2-}{21de^3(e\sin(c+dx)}$$

Mathematica [**A**]

time = 0.67, size = 144, normalized size = 0.75

$$-\frac{2\csc^4(c+dx)\sqrt{e\sin(c+dx)}\left(\frac{1}{4}(36a^2b-2b^3+a(17a^2+30b^2)\cos(c+dx)+14b^3\cos(2(c+dx))-5a^3\cos(3(c+dx))+6ab^2\cos(3(c+dx)))+a(5a^2-6b^2)\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{7}{2}}(c+dx)\right)}{21de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/(e*Sin[c + d*x])^(9/2),x]
```

```
[Out] (-2*Csc[c + d*x]^4*Sqrt[e*Sin[c + d*x]]*((36*a^2*b - 2*b^3 + a*(17*a^2 + 30
*b^2)*Cos[c + d*x] + 14*b^3*Cos[2*(c + d*x)] - 5*a^3*Cos[3*(c + d*x)] + 6*a
*b^2*Cos[3*(c + d*x)])/4 + a*(5*a^2 - 6*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/
4, 2]*Sin[c + d*x]^(7/2)))/(21*d*e^5)
```

Maple [A]
time = 0.16, size = 265, normalized size = 1.37

method	result
default	$-\frac{2b\left(7\left(\cos^2(dx+c)\right)b^2+9a^2-4b^2\right)}{21e\left(e\sin(dx+c)\right)^{\frac{7}{2}}}-\frac{a\left(5\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{9}{2}}(dx+c)\right)\text{EllipticF}\left(\sqrt{-\sin(d\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/21*b/e/(e*sin(d*x+c))^(7/2)*(7*cos(d*x+c)^2*b^2+9*a^2-4*b^2)-1/21*a/e^4
*(5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(9/2)*EllipticF
((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(9/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b
^2-10*a^2*cos(d*x+c)^4*sin(d*x+c)+12*b^2*cos(d*x+c)^4*sin(d*x+c)+16*a^2*cos
(d*x+c)^2*sin(d*x+c)+6*b^2*cos(d*x+c)^2*sin(d*x+c))/sin(d*x+c)^4/cos(d*x+c)
/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] e^(-9/2)*integrate((b*cos(d*x + c) + a)^3/sin(d*x + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 287, normalized size = 1.49

$$\frac{\sqrt{-i}\left(\sqrt{2}\,(5\,a^3-6\,ab^2)\cos(dx+c)^4-2\,\sqrt{2}\,(5\,a^3-6\,ab^2)\cos(dx+c)^2+\sqrt{2}\,(5\,a^3-6\,ab^2)\right)\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{i}\left(\sqrt{2}\,(5\,a^3-6\,ab^2)\cos(dx+c)^4-2\,\sqrt{2}\,(5\,a^3-6\,ab^2)\cos(dx+c)^2+\sqrt{2}\,(5\,a^3-6\,ab^2)\right)\text{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c))-2\left(7\,b^3\cos(dx+c)^2-(5\,a^3-6\,ab^2)\cos(dx+c)^3+9\,a^2b-4\,b^3+(8\,a^3+3\,ab^2)\cos(dx+c)\right)\sqrt{\sin(dx+c)}}{21\left(d\cos(dx+c)^4\,e^{\frac{9}{2}}-2\,d\cos(dx+c)^2\,e^{\frac{9}{2}}+de^{\frac{9}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(-I)*(sqrt(2)*(5*a^3 - 6*a*b^2)*cos(d*x + c)^4 - 2*sqrt(2)*(5*a^3
 - 6*a*b^2)*cos(d*x + c)^2 + sqrt(2)*(5*a^3 - 6*a*b^2))*weierstrassPInverse
(4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(I)*(sqrt(2)*(5*a^3 - 6*a*b^2)*
cos(d*x + c)^4 - 2*sqrt(2)*(5*a^3 - 6*a*b^2)*cos(d*x + c)^2 + sqrt(2)*(5*a^
3 - 6*a*b^2))*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*
(7*b^3*cos(d*x + c)^2 - (5*a^3 - 6*a*b^2)*cos(d*x + c)^3 + 9*a^2*b - 4*b^3
+ (8*a^3 + 3*a*b^2)*cos(d*x + c))*sqrt(sin(d*x + c)))/(d*cos(d*x + c)^4*e^(
9/2) - 2*d*cos(d*x + c)^2*e^(9/2) + d*e^(9/2))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*e^(-9/2)/sin(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\cos\left(c+d\,x\right))^3}{(e\,\sin\left(c+d\,x\right))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(9/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^3/(e*sin(c + d*x))^(9/2), x)
```

3.58 $\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=544

$$\frac{(-a^2+b^2)^{9/4}\,e^{11/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{11/2}d}+\frac{(-a^2+b^2)^{9/4}\,e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{11/2}d}+\frac{2a(2$$

```
[Out] (-a^2+b^2)^(9/4)*e^(11/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(11/2)/d+(-a^2+b^2)^(9/4)*e^(11/2)*arctanh(b^(1/2)*(e*sin(d*
x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/d+2/35*e^3*(7*a^2-7*b^2-5*a*
b*cos(d*x+c))*(e*sin(d*x+c))^(5/2)/b^3/d-2/9*e*(e*sin(d*x+c))^(9/2)/b/d-2/2
1*a*(21*a^4-49*a^2*b^2+33*b^4)*e^6*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(
1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+
c)^(1/2)/b^6/d/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)^3*e^6*(sin(1/2*c+1/4*Pi+1/2
*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*
x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^6/d/(a^2-b*(b-(-a^2
+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)^3*e^6*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x
),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^6/d/(a^2-b*(b+(-a^2+
b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-2/21*e^5*(21*(a^2-b^2)^2-a*b*(7*a^2-12*b^
2)*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^5/d
```

Rubi [A]

time = 1.26, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2774, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{e^{11/2}(b^2-a^2)^{9/4}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}d}+\frac{e^{11/2}(b^2-a^2)^{9/4}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}d}-\frac{ae^6(a^2-b^2)^3\,\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{b^6d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}-\frac{ae^6(a^2-b^2)^3\,\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{b^6d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}-\frac{2e^5\sqrt{e\sin(c+dx)}\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)}{21b^5d}+\frac{2e^3(e\sin(c+dx))^{5/2}\left(7(a^2-b^2)-5ab\cos(c+dx)\right)}{35b^3d}+\frac{2ae^6(21a^4-49a^2b^2+33b^4)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21b^6d\sqrt{e\sin(c+dx)}}-\frac{2e(e\sin(c+dx))^{9/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-a^2 + b^2)^(9/4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 +
 b^2)^(1/4)*Sqrt[e])])/(b^(11/2)*d) + ((-a^2 + b^2)^(9/4)*e^(11/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(11/2)*d)
+ (2*a*(21*a^4 - 49*a^2*b^2 + 33*b^4)*e^6*EllipticF[(c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(21*b^6*d*Sqrt[e*Sin[c + d*x]]) - (a*(a^2 - b^2)^3*e^6*
EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (a
*(a^2 - b^2)^3*e^6*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(b^6*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*
Sin[c + d*x]]) - (2*e^5*(21*(a^2 - b^2)^2 - a*b*(7*a^2 - 12*b^2)*Cos[c + d*
x])*Sqrt[e*Sin[c + d*x]])/(21*b^5*d) + (2*e^3*(7*(a^2 - b^2) - 5*a*b*Cos[c
```

```
+ d*x])*(e*Sin[c + d*x])^(5/2))/(35*b^3*d) - (2*e*(e*Sin[c + d*x])^(9/2))/(
9*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{11/2}}{a+b\cos(c+dx)}\,dx = -\frac{2e(e\sin(c+dx))^{9/2}}{9bd} - \frac{e^2\int \frac{(-b-a\cos(c+dx))(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}dx}{b}$$

$$= \frac{2e^3(7(a^2-b^2)-5ab\cos(c+dx))\,(e\sin(c+dx))^{5/2}}{35b^3d} - \frac{2e(e\sin(c+dx))^{9/2}}{9bd} - \frac{(2\ldots}{}$$

$$= -\frac{2e^5\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21b^5d} + \frac{2e^3(7(a^2-\ldots}{}$$

$$= -\frac{2e^5\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21b^5d} + \frac{2e^3(7(a^2-\ldots}{}$$

$$= -\frac{2e^5\left(21(a^2-b^2)^2-ab(7a^2-12b^2)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21b^5d} + \frac{2e^3(7(a^2-\ldots}{}$$

$$= \frac{2a(21a^4-49a^2b^2+33b^4)\,e^6F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{21b^6d\sqrt{e\sin(c+dx)}} - \frac{2e^5\left(21(a^2-\ldots\right.}{}$$

$$= \frac{2a(21a^4-49a^2b^2+33b^4)\,e^6F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{21b^6d\sqrt{e\sin(c+dx)}} + \frac{a(-a^2+b^2)\ldots}{}$$

$$= \frac{(-a^2+b^2)^{9/4}\,e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{11/2}d} + \frac{(-a^2+b^2)^{9/4}\,e^{11/2}\tanh^{-1}\left(\ldots\right.}{b^{11/2}d\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 47.42, size = 2035, normalized size = 3.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (((a*(28*a^2 - 51*b^2)*Cos[c + d*x])/(42*b^4) + ((-9*a^2 + 14*b^2)*Cos[2*(c
 + d*x)])/(45*b^3) + (a*Cos[3*(c + d*x)])/(14*b^2) - Cos[4*(c + d*x)]/(36*b
))*Csc[c + d*x]^5*(e*Sin[c + d*x])^(11/2))/d - ((e*Sin[c + d*x])^(11/2)*((2
*(392*a^3*b - 722*a*b^3)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((
a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] +
```

```
2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[
Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*
Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqr
t[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) +
 (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c +
d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^
2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(
-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Si
n[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c
+ d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-
1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(-
280*a^4 + 636*a^2*b^2 - 721*b^4)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^
2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]
])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/
(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(
1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2
 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2,
 (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d
*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c
 + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]
^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1,
 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(
a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c +
d*x]^2]) + ((840*a^4 - 1764*a^2*b^2 + 959*b^4)*Cos[c + d*x]*Cos[2*(c + d*x)
]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 -
((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 +
b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Sin[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + ((1/4 -
I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - (
(1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/
4)) + (4*Sqrt[Sin[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)
) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c
 + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/
(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Si
n[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c
 + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(
-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[
1 - Sin[c + d*x]^2])))/(1680*b^4*d*Sin[c + d*x]^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2047 vs.

2(569) = 1138.
time = 0.32, size = 2048, normalized size = 3.76

method	result	size
default	Expression too large to display	2048

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/9/b*e*(e*sin(d*x+c))^(9/2)+2/5/b^3*e^3*(e*sin(d*x+c))^(5/2)*a^2-2/5/b*e
^3*(e*sin(d*x+c))^(5/2)-2/b^5*e^5*a^4*(e*sin(d*x+c))^(1/2)+4/b^3*e^5*a^2*(e
*sin(d*x+c))^(1/2)-2/b*e^5*(e*sin(d*x+c))^(1/2)+1/2/b^5*e^7*(e^2*(a^2-b^2)/
b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)+1)*a^6-3/2/b^3*e^7*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e
^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))
^(1/2)+1)*a^4+3/2/b*e^7*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)
*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)*a^2-1/2*b
*e^7*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+1/2/b^5*e^7*(e^2*(a^2-b^2)/b
^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4
)*(e*sin(d*x+c))^(1/2)-1)*a^6-3/2/b^3*e^7*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^
2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^
(1/2)-1)*a^4+3/2/b*e^7*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*
arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)*a^2-1/2*b*
e^7*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2
*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+1/4/b^5*e^7*(e^2*(a^2-b^2)/b^
2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e
^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1
/2)))*a^6-3/4/b^3*e^7*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*l
n((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2
*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c
))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))*a^4+3/4/b*e^7*(e^2*(a^2-b^2)/b
^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1
/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(
e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(
1/2)))*a^2-1/4*b*e^7*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln
((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*
(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c)
)^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)*e^6*a*(-1/21/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(-6*b^4*cos(d*x+c)^4*
sin(d*x+c)+21*a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-49*a^2*b^2*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(
```

```
1/2),1/2*2^(1/2))+33*b^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-14*a^2*b^2*cos(d*x
+c)^2*sin(d*x+c)+30*b^4*cos(d*x+c)^2*sin(d*x+c))+(-a^6+3*a^4*b^2-3*a^2*b^4+
b^6)/b^6*(-1/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/
b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1
/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticP
i((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/
(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] e^(11/2)*integrate(sin(d*x + c)^(11/2)/(b*cos(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(11/2)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate(e^(11/2)*sin(d*x + c)^(11/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\sin\left(c+d\,x\right))^{11/2}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x)), x)
```

3.59 $\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=461

$$-\frac{(-a^2+b^2)^{7/4}\,e^{9/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{9/2}d}+\frac{(-a^2+b^2)^{7/4}\,e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{9/2}d}+\frac{a(a}{}$$

```
[Out] -(-a^2+b^2)^(7/4)*e^(9/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(9/2)/d+(-a^2+b^2)^(7/4)*e^(9/2)*arctanh(b^(1/2)*(e*sin(d*x+
c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/d+2/15*e^3*(5*a^2-5*b^2-3*a*b*c
os(d*x+c))*(e*sin(d*x+c))^(3/2)/b^3/d-2/7*e*(e*sin(d*x+c))^(7/2)/b/d-a*(a^2
-b^2)^2*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d
*x+c)^(1/2)/b^5/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-a*(a^2-b^2)^2*e
^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi
(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/
2)/b^5/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2/5*a*(5*a^2-8*b^2)*e^4*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^4/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.84, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2774, 2944, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{e^{9/2}(b^2-a^2)^{7/4}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}d}+\frac{e^{9/2}(b^2-a^2)^{7/4}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}d}+\frac{ae^5(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{b^5d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{ae^5(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{b^5d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}-\frac{2ae^4(5a^2-8b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}}+\frac{2e^3(e\sin(c+dx))^{3/2}(5(a^2-b^2)-3ab\cos(c+dx))}{15b^3d}-\frac{2e(e\sin(c+dx))^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -(((-a^2 + b^2)^(7/4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(b^(9/2)*d)) + ((-a^2 + b^2)^(7/4)*e^(9/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(9/2)*d) +
 (a*(a^2 - b^2)^2*e^5*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^5*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c +
 d*x]]) + (a*(a^2 - b^2)^2*e^5*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c
- Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^5*(b + Sqrt[-a^2 + b^2])*d*Sqrt[
e*Sin[c + d*x]]) - (2*a*(5*a^2 - 8*b^2)*e^4*EllipticE[(c - Pi/2 + d*x)/2, 2
]*Sqrt[e*Sin[c + d*x]])/(5*b^4*d*Sqrt[Sin[c + d*x]]) + (2*e^3*(5*(a^2 - b^2
) - 3*a*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*b^3*d) - (2*e*(e*Sin[c
+ d*x])^(7/2))/(7*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{9/2}}{a+b\cos(c+dx)}\,dx = -\frac{2e(e\sin(c+dx))^{7/2}}{7bd} - \frac{e^2\int \frac{(-b-a\cos(c+dx))(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx}{b}$$

$$= \frac{2e^3(5(a^2-b^2)-3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15b^3d} - \frac{2e(e\sin(c+dx))^{7/2}}{7bd} - \frac{(2e \ldots}{\ldots}$$

$$= \frac{2e^3(5(a^2-b^2)-3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15b^3d} - \frac{2e(e\sin(c+dx))^{7/2}}{7bd} - \frac{(a(\ldots}{\ldots}$$

$$= \frac{2e^3(5(a^2-b^2)-3ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{15b^3d} - \frac{2e(e\sin(c+dx))^{7/2}}{7bd} - \frac{\left(a(\ldots\right.}{\ldots}$$

$$= -\frac{2a(5a^2-8b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}} + \frac{2e^3(5(a^2-b^2)-3ab \ldots}{\ldots}$$

$$= \frac{a(a^2-b^2)^2\,e^5\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{a(a^2-b^2)^2 \ldots}{\ldots}$$

$$= -\frac{(-a^2+b^2)^{7/4}\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{9/2}d} + \frac{(-a^2+b^2)^{7/4}\,e^{9/2}\tanh^{-1}\left(\ldots\right.}{b^{9/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 35.04, size = 834, normalized size = 1.81

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -1/5*((e*Sin[c + d*x])^(9/2)*(((5*a^3 - 8*a*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*
a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2
 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b
^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[S
in[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/
4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c +
 d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a
```

```
+ b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(2*a^2*b - 5*b^3)*Cos[c + d*x]
*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b
^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)
^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-
a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 +
b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*
x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Si
n[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(b^3*d*Si
n[c + d*x]^(9/2)) + (Csc[c + d*x]^4*(e*Sin[c + d*x])^(9/2)*(-1/42*((-28*a^2
 + 37*b^2)*Sin[c + d*x])/b^3 - (a*Sin[2*(c + d*x)])/(5*b^2) + Sin[3*(c + d*
x)]/(14*b)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. $2(494) = 988$.

time = 0.27, size = 1473, normalized size = 3.20

method	result	size
default	Expression too large to display	1473

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/7*e/b*(e*sin(d*x+c))^(7/2)+2/3*e^3/b^3*(e*sin(d*x+c))^(3/2)*a^2-2/3*e^3
/b*(e*sin(d*x+c))^(3/2)-1/2*e^5/b^5/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)*a^4+e^5/b^3/(e^
2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*
sin(d*x+c))^(1/2)-1)*a^2-1/2*e^5/b/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)-1/4*e^5/b^5/(e^2
*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e
*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a
^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))
*a^4+1/2*e^5/b^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln((e*sin(d*x+c)-(e^2*(a
^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/
(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(
a^2-b^2)/b^2)^(1/2)))*a^2-1/4*e^5/b/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln((e
*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^
2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(
1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-1/2*e^5/b^5/(e^2*(a^2-b^2)/b^2)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1
)*a^4+e^5/b^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)*a^2-1/2*e^5/b/(e^2*(a^2-b^2)/b^2)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)
+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^5*a*(1/5/b^4/(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)*(10*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*
```

```
EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-16*(-sin(d*x+c)+1)^(1/2)*(
2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*
2^(1/2))*b^2-5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+8*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2
*2^(1/2))*b^2+2*b^2*cos(d*x+c)^4-2*cos(d*x+c)^2*b^2)+(a^4-2*a^2*b^2+b^4)/b^
4*(-1/2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*
x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(-sin(d*x+c)+1)
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^
(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^
2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] e^(9/2)*integrate(sin(d*x + c)^(9/2)/(b*cos(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(9/2)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate(e^(9/2)*sin(d*x + c)^(9/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\sin(c+d\,x))^{9/2}}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x)), x)
```

3.60 $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=474

$$\frac{(-a^2+b^2)^{5/4}\, e^{7/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{7/2}d}+\frac{(-a^2+b^2)^{5/4}\, e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{7/2}d}-\frac{2a(3}{}$$

```
[Out] (-a^2+b^2)^(5/4)*e^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/
4)/e^(1/2))/b^(7/2)/d+(-a^2+b^2)^(5/4)*e^(7/2)*arctanh(b^(1/2)*(e*sin(d*x+c
))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/d-2/5*e*(e*sin(d*x+c))^(5/2)/b/d
+2/3*a*(3*a^2-4*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/b
^4/d/(e*sin(d*x+c))^(1/2)-a*(a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-
(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/(a^2-b*(b-(-a^2+b^2)^(1/2
)))/(e*sin(d*x+c))^(1/2)-a*(a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(
-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/(a^2-b*(b+(-a^2+b^2)^(1/2)
))/(e*sin(d*x+c))^(1/2)+2/3*e^3*(3*a^2-3*b^2-a*b*cos(d*x+c))*(e*sin(d*x+c))
^(1/2)/b^3/d
```

Rubi [A]

time = 0.86, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2774, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{e^{7/2}(b^2-a^2)^{5/4}\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}d}+\frac{e^{7/2}(b^2-a^2)^{5/4}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}d}-\frac{2ae^4(3a^2-4b^2)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3b^4d\sqrt{e\sin(c+dx)}}+\frac{ae^4(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{b^4d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}+\frac{ae^4(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{b^4d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}+\frac{2e^3\sqrt{e\sin(c+dx)}\,(3(a^2-b^2)-ab\cos(c+dx))}{3b^3d}-\frac{2e(e\sin(c+dx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-a^2 + b^2)^(5/4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(7/2)*d) + ((-a^2 + b^2)^(5/4)*e^(7/2)*ArcTanh[(Sq
rt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(7/2)*d) - (2
*a*(3*a^2 - 4*b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])
/(3*b^4*d*Sqrt[e*Sin[c + d*x]]) + (a*(a^2 - b^2)^2*e^4*EllipticPi[(2*b)/(b
- Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^4*(a^2 -
 b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (a*(a^2 - b^2)^2*e^4*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (2*
e^3*(3*(a^2 - b^2) - a*b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*b^3*d) - (2
*e*(e*Sin[c + d*x])^(5/2))/(5*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx = -\frac{2e(e\sin(c+dx))^{5/2}}{5bd} - \frac{e^2\int \frac{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx}{b}$$

$$= \frac{2e^3(3(a^2-b^2)-ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd} - \frac{(2e^4)\int \cdots}{\cdots}$$

$$= \frac{2e^3(3(a^2-b^2)-ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd} - \frac{(a(3a^2 \cdots}{\cdots}$$

$$= \frac{2e^3(3(a^2-b^2)-ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} - \frac{2e(e\sin(c+dx))^{5/2}}{5bd} - \frac{\left(a(-a \cdots\right.}{\cdots}$$

$$= -\frac{2a(3a^2-4b^2)\,e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}} + \frac{2e^3(3(a^2-b^2)-ab\cos(\cdots}{3b\cdots}$$

$$= -\frac{2a(3a^2-4b^2)\,e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}} + \frac{a(-a^2+b^2)^{3/2}\,e^4\Pi\left(\frac{\cdots}{b-\cdots}\right.}{b^4\left(b-\cdots\right.}$$

$$= \frac{(-a^2+b^2)^{5/4}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{7/2}d} + \frac{(-a^2+b^2)^{5/4}\,e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt[4]{\cdots}}\right.}{b^{7/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 45.91, size = 1955, normalized size = 4.12

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (((-2*a*Cos[c + d*x])/(3*b^2) + Cos[2*(c + d*x)]/(5*b))*Csc[c + d*x]^3*(e*S
in[c + d*x])^(7/2))/d + ((e*Sin[c + d*x])^(7/2)*((28*a*b*Cos[c + d*x]^2*(a
+ b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[
c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c +
d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^
2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/
```

```
4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sq
rt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c
+ d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2,
 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*A
ppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)
])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x
])*(1 - Sin[c + d*x]^2)) + (2*(-10*a^2 + 27*b^2)*Cos[c + d*x]*(a + b*Sqrt[1
 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*S
qrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt
[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b
]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2
 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c
 + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4,
 Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sq
rt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9
/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*Appel
lF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*S
in[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*S
qrt[1 - Sin[c + d*x]^2]) + ((30*a^2 - 33*b^2)*Cos[c + d*x]*Cos[2*(c + d*x)]
*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - (
(1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b
^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[S
in[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + ((1/4 - I
/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((
1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^
2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4
)) + (4*Sqrt[Sin[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x
]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2))
 + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c
+ d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a
^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(
-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin
[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c
+ d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-
1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1
 - Sin[c + d*x]^2])))/(60*b^2*d*Sin[c + d*x]^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1533 vs. $2(507) = 1014$.
time = 0.24, size = 1534, normalized size = 3.24

method	result	size
default	Expression too large to display	1534

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/5/b*e*(e*sin(d*x+c))^(5/2)+2/b^3*e^3*a^2*(e*sin(d*x+c))^(1/2)-2/b*e^3*(
e*sin(d*x+c))^(1/2)-1/2/b^3*e^5*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)
*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)*a
^4+1/b*e^5*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/
2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)*a^2-1/2*b*e^5*(e^2*(a^
2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b
^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)-1/4/b^3*e^5*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^
2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*
x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))*a^4+1/
2/b*e^5*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c
)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2
)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/
2)+(e^2*(a^2-b^2)/b^2)^(1/2)))*a^2-1/4*b*e^5*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2
*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x
+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/
b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-1/2/b^3
*e^5*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)*a^4+1/b*e^5*(e^2*(a^2-b^2)/b
^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4
)*(e*sin(d*x+c))^(1/2)+1)*a^2-1/2*b*e^5*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-
b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1
/2)+1)+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^4*a*(1/3/b^4/(cos(d*x+c)^2*e*sin
(d*x+c))^(1/2)*(3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-4*(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))*b^2-2*b^2*cos(d*x+c)^2*sin(d*x+c))+(a^4-2*a^2*b^2+b^4)/b^4*(-1
/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticP
i((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b
^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+
c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c
))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] e^(7/2)*integrate(sin(d*x + c)^(7/2)/(b*cos(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*cos(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate(e^(7/2)*sin(d*x + c)^(7/2)/(b*cos(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\sin\left(c+d\,x\right))^{7/2}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x)), x)
```

3.61 $\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=399

$$-\frac{(-a^2+b^2)^{3/4}e^{5/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d}+\frac{(-a^2+b^2)^{3/4}e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d}-\frac{a(a}{}$$

```
[Out] -(-a^2+b^2)^(3/4)*e^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(5/2)/d+(-a^2+b^2)^(3/4)*e^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+
c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/d-2/3*e*(e*sin(d*x+c))^(3/2)/b/
d+a*(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))
*sin(d*x+c)^(1/2)/b^3/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+a*(a^2-b^
2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)
^(1/2)/b^3/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*a*e^2*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*P
i+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^2/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2774, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{e^{5/2}(b^2-a^2)^{3/4}\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d}+\frac{e^{5/2}(b^2-a^2)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}d}-\frac{ae^3(a^2-b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{ae^3(a^2-b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{b^3d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}+\frac{2ae^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}}-\frac{2e(e\sin(c+dx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -(((-a^2 + b^2)^(3/4)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(b^(5/2)*d)) + ((-a^2 + b^2)^(3/4)*e^(5/2)*ArcTanh[
(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(5/2)*d) -
 (a*(a^2 - b^2)*e^3*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^3*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d
*x]]) - (a*(a^2 - b^2)*e^3*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi
/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^3*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Si
n[c + d*x]]) + (2*a*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x
]])/(b^2*d*Sqrt[Sin[c + d*x]]) - (2*e*(e*Sin[c + d*x])^(3/2))/(3*b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
```

```
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx = -\frac{2e(e\sin(c+dx))^{3/2}}{3bd} - \frac{e^2\int \frac{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{b}$$

$$= -\frac{2e(e\sin(c+dx))^{3/2}}{3bd} + \frac{(ae^2)\int \sqrt{e\sin(c+dx)}\,dx}{b^2} - \frac{((a^2-b^2)e^2)\int \frac{\sqrt{e\sin(c}}{a+b\cos(c}}{b^2}$$

$$= -\frac{2e(e\sin(c+dx))^{3/2}}{3bd} + \frac{(a(a^2-b^2)e^3)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right.}}{2b^3}$$

$$= \frac{2ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}} - \frac{2e(e\sin(c+dx))^{3/2}}{3bd} + \frac{(2(a^2-b^2)e}{}$$

$$= -\frac{a(a^2-b^2)e^3\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{a(a^2-b^2)}{}$$

$$= -\frac{(-a^2+b^2)^{3/4}e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{5/2}d} + \frac{(-a^2+b^2)^{3/4}e^{5/2}\tanh^{-1}\left(\right.}{b^{5/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 34.45, size = 757, normalized size = 1.90

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*Csc[c + d*x]*(e*Sin[c + d*x])^(5/2))/(3*b*d) + ((e*Sin[c + d*x])^(5/2)*
((a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sq
rt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b
]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sq
rt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2
 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c +
d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c +
 d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2])
)/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*
b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*
```

```
x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]]
)/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)
^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sq
rt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (
b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a +
 b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2
])))/(b*d*Sin[c + d*x]^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(435) = 870$.

time = 0.21, size = 968, normalized size = 2.43

method	result	size
default	Expression too large to display	968

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/3*e/b*(e*sin(d*x+c))^(3/2)+1/2*e^3/b^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)*a^2-1/2*
e^3/b/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^
(1/4)*(e*sin(d*x+c))^(1/2)-1)+1/4*e^3/b^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)
*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e
^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x
+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))*a^2-1/4*e^3/b/(e^2*(a^2-b^2)
/b^2)^(1/4)*2^(1/2)*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c
))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^
2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+1/2*e^3/b
^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)+1)*a^2-1/2*e^3/b/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+(cos(d*x+c
)^2*e*sin(d*x+c))^(1/2)*e^3*a*(-1/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(2*EllipticE((-s
in(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)
))-(a^2-b^2)/b^2*(-1/2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*Elli
pticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*
(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2
),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/
d
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate(sin(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(e^(5/2)*sin(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,\sin\left(c+d\,x\right)\right)^{5/2}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x)),x)
```

```
[Out] int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x)), x)
```

3.62 $\int \frac{(e \sin(c+dx))^{3/2}}{a+b \cos(c+dx)} \, dx$

Optimal. Leaf size=410

$$\frac{\sqrt[4]{-a^2+b^2}\, e^{3/2} \text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e \sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{3/2} d}+\frac{\sqrt[4]{-a^2+b^2}\, e^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e \sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{b^{3/2} d}+\frac{2ae^2 F\left(\frac{1}{2}\right.}{}$$

```
[Out] (-a^2+b^2)^(1/4)*e^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/
4)/e^(1/2))/b^(3/2)/d+(-a^2+b^2)^(1/4)*e^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c
))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/d-2*a*e^2*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x)
,2^(1/2))*sin(d*x+c)^(1/2)/b^2/d/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)*e^2*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2
*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/d
/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+a*(a^2-b^2)*e^2*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+
1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/d/(a
^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-2*e*(e*sin(d*x+c))^(1/2)/b/
d
```

Rubi [A]
time = 0.58, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2774, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{e^{3/2}\sqrt[4]{b^2-a^2}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}d}-\frac{ae^2(a^2-b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}-\frac{ae^2(a^2-b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{b^2d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}+\frac{e^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}d}+\frac{2ae^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{b^2d\sqrt{e\sin(c+dx)}}-\frac{2e\sqrt{e\sin(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-a^2 + b^2)^(1/4)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*d) + ((-a^2 + b^2)^(1/4)*e^(3/2)*ArcTanh[(Sq
rt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*d) + (2
*a*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^2*d*Sqrt[e*S
in[c + d*x]]) - (a*(a^2 - b^2)*e^2*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
 (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^2*(a^2 - b*(b - Sqrt[-a^2 +
b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (a*(a^2 - b^2)*e^2*EllipticPi[(2*b)/(b + S
qrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^2*(a^2 - b*
(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (2*e*Sqrt[e*Sin[c + d*x]]
)/(b*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx = -\frac{2e\sqrt{e\sin(c+dx)}}{bd} - \frac{e^2\int \frac{-b-a\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{b}$$

$$= -\frac{2e\sqrt{e\sin(c+dx)}}{bd} + \frac{(ae^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{b^2} + \frac{((-a^2+b^2)\,e^2)\int \frac{}{(a+b\cos(c+}}{b^2}$$

$$= -\frac{2e\sqrt{e\sin(c+dx)}}{bd} - \frac{\left(a\sqrt{-a^2+b^2}\,e^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+\right.}}{2b^2}$$

$$= \frac{2ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{b^2d\sqrt{e\sin(c+dx)}} - \frac{2e\sqrt{e\sin(c+dx)}}{bd} + \frac{(2(a^2-b^2)\,e^3)}{}$$

$$= \frac{2ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{b^2d\sqrt{e\sin(c+dx)}} + \frac{a\sqrt{-a^2+b^2}\,e^2\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(\right.}{b^2\left(b-\sqrt{-a^2+b^2}\right)}$$

$$= \frac{\sqrt[4]{-a^2+b^2}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{b^{3/2}d} + \frac{\sqrt[4]{-a^2+b^2}\,e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{}}{\sqrt[4]{-}}\right.}{b^{3/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 15.87, size = 434, normalized size = 1.06

$$-\frac{\left(\frac{1}{20}-\frac{i}{20}\right)\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)(e\sin(c+dx))^{3/2}\left(-5(a^2-b^2)\left(2\sqrt[4]{-a^2+b^2}\,\mathrm{ArcTan}\left(1-\frac{(1+i)\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-2\sqrt[4]{-a^2+b^2}\,\mathrm{ArcTan}\left(1+\frac{(1+i)\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)+\sqrt[4]{-a^2+b^2}\log\left(\sqrt{-a^2+b^2}-(1+i)\sqrt{b}\sqrt[4]{-a^2+b^2}\sqrt{\sin(c+dx)}+ib\sin(c+dx)\right)-\sqrt[4]{-a^2+b^2}\log\left(\sqrt{-a^2+b^2}+(1+i)\sqrt{b}\sqrt[4]{-a^2+b^2}\sqrt{\sin(c+dx)}+ib\sin(c+dx)\right)+(4+4i)\sqrt{b}\sqrt{\sin(c+dx)}\right)+(4+4i)ab^{3/2}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\sin^2(c+dx),\frac{b^2\sin^2(c+dx)}{-a^2+b^2}\right)\sin^{\frac{5}{2}}(c+dx)\right)}{b^{3/2}(-a^2+b^2)\,d\sqrt{\cos^2(c+dx)}\,(a+b\cos(c+dx))\sin^{\frac{3}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-1/20 + I/20)*Cos[c + d*x]*(a + b*Sqrt[Cos[c + d*x]^2])*(e*Sin[c + d*x])^
(3/2)*(-5*(a^2 - b^2)*(2*(-a^2 + b^2)^(1/4)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqr
t[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*(-a^2 + b^2)^(1/4)*ArcTan[1 + ((1
+ I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + (-a^2 + b^2)^(1/4)*L
og[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
 + I*b*Sin[c + d*x]] - (-a^2 + b^2)^(1/4)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + (4 + 4*I)
*Sqrt[b]*Sqrt[Sin[c + d*x]]) + (4 + 4*I)*a*b^(3/2)*AppellF1[5/4, 1/2, 1, 9/
4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(5/2)))/
(b^(3/2)*(-a^2 + b^2)*d*Sqrt[Cos[c + d*x]^2]*(a + b*Cos[c + d*x])*Sin[c + d
*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(448) = 896$.
time = 0.20, size = 1068, normalized size = 2.60

method	result	size
default	Expression too large to display	1068

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*e/b*(e*sin(d*x+c))^(1/2)+1/4*e^3/b*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b
^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(
1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(
1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))*a^2-1/4*e^3*b
*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*
(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)
)/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2
*(a^2-b^2)/b^2)^(1/2)))+1/2*e^3/b*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^
2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)
*a^2-1/2*e^3*b*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2
^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+1/2*e^3/b*(e^2*(a^
2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b
^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)*a^2-1/2*e^3*b*(e^2*(a^2-b^2)/b^2)^(1/4)/(
a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*
x+c))^(1/2)-1)+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^2*a*(-1/b^2*(-sin(d*x+c)
+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+(-a^2+b^2)/b^2*(-1/2/
b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((
-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^2)
^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+
1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^
(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(sin(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^{\frac{3}{2}}}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*cos(d*x+c)),x)
[Out] Integral((e*sin(c + d*x))**(3/2)/(a + b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate(e^(3/2)*sin(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\sin(c+d\,x))^{3/2}}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x)), x)
```

3.63 $\int \frac{\sqrt{e \sin(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=302

$$-\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{\sqrt{b}\,\sqrt[4]{-a^2+b^2}\,d}+\frac{\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{\sqrt{b}\,\sqrt[4]{-a^2+b^2}\,d}+\frac{ae\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{b\left(b-\sqrt{-a^2+b^2}\right)d}$$

```
[Out] -arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^
2+b^2)^(1/4)/d/b^(1/2)+arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4
)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(1/4)/d/b^(1/2)-a*e*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2
*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/d/(b-(-a^2+b^2)^(1/2))/
(e*sin(d*x+c))^(1/2)-a*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2
^(1/2))*sin(d*x+c)^(1/2)/b/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}\,d\sqrt[4]{b^2-a^2}}+\frac{\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}\,d\sqrt[4]{b^2-a^2}}+\frac{ae\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)}{bd\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{ae\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)}{bd\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] -((Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(Sqrt[b]*(-a^2 + b^2)^(1/4)*d)) + (Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Sin
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(Sqrt[b]*(-a^2 + b^2)^(1/4)*d) +
 (a*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/(b*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*e*Ell
ipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(b*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx = -\frac{(ae)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2b} + \frac{(ae)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-\right.}}}{2b}$$

$$= -\frac{(2be)\mathrm{Subst}\left(\int \frac{x^2}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} - \frac{\left(ae\sqrt{\sin(c+dx)}\right)\int \frac{}{\sqrt{s}}}{2b}$$

$$= \frac{ae\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}}; \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{b\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{ae\Pi\left(\frac{2b}{b+\sqrt{-a^2+b^2}}; \frac{1}{2}(\right.}{b\left(b+\sqrt{-a^2+\right.}}$$

$$= -\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}\,d} + \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}\,d} + \frac{aeI}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 11.80, size = 361, normalized size = 1.20

$$\frac{2\cos(c+dx)\left(a+b\sqrt{\cos^2(c+dx)}\right)\sqrt{e\sin(c+dx)}\left(\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(2\mathrm{ArcTan}\left(1-\frac{(1+i)\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-2\mathrm{ArcTan}\left(1+\frac{(1+i)\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-\log\left(\sqrt{-a^2+b^2}-(1+i)\sqrt{b}\sqrt[4]{-a^2+b^2}\sqrt{\sin(c+dx)}+ib\sin(c+dx)\right)+\log\left(\sqrt{-a^2+b^2}+(1+i)\sqrt{b}\sqrt[4]{-a^2+b^2}\sqrt{\sin(c+dx)}+ib\sin(c+dx)\right)\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}}+\frac{aF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\sin^2(c+dx),\frac{b^2\sin^2(c+dx)}{-a^2+b^2}\right)\sin^{\frac{3}{2}}(c+dx)}{3(a^2-b^2)}\right)}{d\sqrt{\cos^2(c+dx)}\,(a+b\cos(c+dx))\sqrt{\sin(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[c + d*x]*(a + b*Sqrt[Cos[c + d*x]^2])*Sqrt[e*Sin[c + d*x]]*(((1/8 +
I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] -
Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]
] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)
^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)
) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^
2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2))))/(d*Sqrt[Cos[c + d*x]^2]*(a
+ b*Cos[c + d*x])*Sqrt[Sin[c + d*x]])
```

Maple [A]
time = 0.16, size = 545, normalized size = 1.80

method	result

default	$-\frac{e\sqrt{2}\,\ln\left(\frac{e\sin(dx+c)-\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2(a^2-b^2)}{b^2}}}{e\sin(dx+c)+\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2(a^2-b^2)}{b^2}}}\right)}{4b\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}-\frac{e\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c}}{\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}\right.}{2b\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*e/b/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(
d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2)))-1/2*e/b/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)-1/2*e/b/(e^2*(a^2-b^2)/b^2)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
-1)+1/2*e*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/b
*(EllipticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2))*(-a
^2+b^2)^(1/2)+EllipticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*
2^(1/2))*b-EllipticPi((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1
/2))*(-a^2+b^2)^(1/2)+EllipticPi((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2
)),1/2*2^(1/2))*b)/((-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2))/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(sin(d*x + c))/(b*cos(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \sin(c+dx)}}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*cos(d*x+c)),x)
[Out] Integral(sqrt(e*sin(c + d*x))/(a + b*cos(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] integrate(e^(1/2)*sqrt(sin(d*x + c))/(b*cos(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,\sin\left(c+d\,x\right)}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x)), x)
```

3.64 $\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=307

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}\,d\sqrt{e}}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}\,d\sqrt{e}}+\frac{a\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+\right.\right.}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right.}$$

[Out] arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(3/4)/d/e^(1/2)+arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(3/4)/d/e^(1/2)-a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)

Rubi [A]
time = 0.37, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\,(b^2-a^2)^{3/4}}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{d\sqrt{e}\,(b^2-a^2)^{3/4}}+\frac{a\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}+\frac{a\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]

[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(3/4)*d*Sqrt[e]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(3/4)*d*Sqrt[e]) + (a*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (a*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]])

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx = -\frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{2\sqrt{-a^2+b^2}} - \frac{a\int \frac{}{\sqrt{e\sin}}}{}$$

$$= -\frac{(2be)\text{Subst}\left(\int \frac{1}{(a^2-b^2)e^2+b^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} - \frac{\left(a\sqrt{\sin}\right.}{}$$

$$= \frac{a\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}}; \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right) \middle| 2\right)\sqrt{\sin(c+dx)}}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)d\sqrt{e\sin(c+dx)}} + \frac{a\Pi\left(\right.}{\left(\right.}$$

$$= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{3/4}d\sqrt{e}} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin}}{\sqrt[4]{-a^2+}}\right.}{(-a^2+b^2)^{3/4}d\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 12.03, size = 261, normalized size = 0.85

$$\frac{10(a+b)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(-a+b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)\sqrt{e\sin(c+dx)}}{de(a+b\cos(c+dx))\left(5(a+b)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(-a+b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)+2\left(-2(a-b)F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(-a+b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)+(a+b)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(-a+b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)\right)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (10*(a + b)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[
(c + d*x)/2]^2)/(a + b)]*Sqrt[e*Sin[c + d*x]])/(d*e*(a + b*Cos[c + d*x])*(5
*(a + b)*AppellF1[1/4, -1/2, 1, 5/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c
+ d*x)/2]^2)/(a + b)] + 2*(-2*(a - b)*AppellF1[5/4, -1/2, 2, 9/4, -Tan[(c +
 d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[5/4,
1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])*T
an[(c + d*x)/2]^2))
```

Maple [A]
time = 0.17, size = 600, normalized size = 1.95

method	result

default	$-\frac{be\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{e\sin(dx+c)+\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}{e\sin(dx+c)-\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2\left(a^2-b^2\right)}{b^2}}}\right)}{4\left(a^2e^2-b^2e^2\right)}-\frac{be\left(\frac{e^2\left(a^2-b^2\right)}{b^2}\right)^{\frac{1}{4}}\sqrt{2}\ a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*b*e*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x
+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b
^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(
1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-1/2*b*e*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2
-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(
1/2)+1)-1/2*b*e*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(
2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+1/2*a*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*(EllipticPi((-sin(d*x+c)
+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2))*(-a^2+b^2)^(1/2)+EllipticPi(
(-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2))*b+EllipticPi((-s
in(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2)-Ell
ipticPi((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b)/(-a^2+
b^2)^(1/2)/((-a^2+b^2)^(1/2)-b)/(b+(-a^2+b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+
c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(1/((b*cos(d*x + c) + a)*sqrt(sin(d*x + c))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/((b*cos(d*x + c)*e^(1/2) + a*e^(1/2))*sqrt(sin(d*x + c))), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\sin{(c+dx)}}\,(a+b\cos{(c+dx)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(e^(-1/2)/((b*cos(d*x + c) + a)*sqrt(sin(d*x + c))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,\sin{(c+d\,x)}}\,(a+b\,\cos{(c+d\,x)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))),x)
[Out] int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))), x)
```

3.65 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=426

$$-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}de^{3/2}}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}de^{3/2}}+\frac{2(b-a\cos(c+dx))}{(a^2-b^2)de\sqrt{e\sin(c+dx)}}-\cdots$$

```
[Out] -b^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^
2+b^2)^(5/4)/d/e^(3/2)+b^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(5/4)/d/e^(3/2)+2*(b-a*cos(d*x+c))/(a^2-b^2)/
d/e/(e*sin(d*x+c))^(1/2)+a*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+
1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2
)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+
c))^(1/2)+a*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin
(d*x+c)^(1/2)/(a^2-b^2)/d/e/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*a*(
sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(
1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)/d/e^2/sin(d*x
+c)^(1/2)
```

Rubi [A]
time = 0.63, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2775, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{3/2}(b^2-a^2)^{5/4}}-\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2(a^2-b^2)\sqrt{\sin(c+dx)}}+\frac{2(b-a\cos(c+dx))}{de(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{ab\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{de(a^2-b^2)\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{ab\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{de(a^2-b^2)\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{3/2}(b^2-a^2)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -((b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/((-a^2 + b^2)^(5/4)*d*e^(3/2))) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(5/4)*d*e^(3/2)) +
 (2*(b - a*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) - (a*b*Ell
ipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (a*b*
EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (2
*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)*d*e^
2*Sqrt[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx = \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{2\int \frac{\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{1}{2}ab\cos(c+dx)\right)\sqrt{e\sin}}{a+b\cos(c+dx)}}{(a^2-b^2)\,e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{a\int \sqrt{e\sin(c+dx)}\,dx}{(a^2-b^2)\,e^2} - \frac{b^2\int}{}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} + \frac{(ab)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2}\right.}}{2\,(a^2-b^2)\,e}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin}}{(a^2-b^2)\,de^2\sqrt{\sin(c+d}}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{ab\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right.}$$

$$= -\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{5/4}\,de^{3/2}} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\,s}}{\sqrt[4]{-a^2}}\right.}{(-a^2+b^2)^{5/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 34.77, size = 791, normalized size = 1.86

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-2*(-b + a*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)*d*(e*Sin[c + d*x])^(3/
2)) - (Sin[c + d*x]^(3/2)*((a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)
*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*
ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sq
rt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Si
n[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[
Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Si
n[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b
*Sqrt[1 - Sin[c + d*x]^2]))/(12*Sqrt[b]*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(
1 - Sin[c + d*x]^2)) + (2*(a^2 + b^2)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[
```

```
1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 +
 ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 +
 b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c +
 d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin
[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1
[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c
 + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*C
os[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/((a - b)*(a + b)*d*(e*Sin[c + d*x]
)^(3/2))
```

Maple [A]
time = 0.18, size = 843, normalized size = 1.98

method	result
default	$\frac{b\sqrt{2}\,\ln\left(\frac{e\sin(dx+c)-\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2(a^2-b^2)}{b^2}}}{e\sin(dx+c)+\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}\sqrt{e\sin(dx+c)}\,\sqrt{2}+\sqrt{\frac{e^2(a^2-b^2)}{b^2}}}\right)}{4e(a-b)(a+b)\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}+\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}-\right.}{2e(a-b)(a+b)\left(\frac{e^2(a^2-b^2)}{b^2}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*b/e/(a-b)/(a+b)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln((e*sin(d*x+c)-(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/
2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e
^2*(a^2-b^2)/b^2)^(1/2)))+1/2*b/e/(a-b)/(a+b)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+1/2*b
/e/(a-b)/(a+b)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b
^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+2*b/e/(a^2-b^2)/(e*sin(d*x+c))^(1/2)
-1/2*a*((-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^
(1/2))*b-(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^
(1/2))*b+(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elli
pticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2))*b^2+(-sin
(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((-sin(d
*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^2+4*(-sin(d*x+c)+1)^(1
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2)
,1/2*2^(1/2))*a^2-2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-4*a^2*cos(d*x+c)^2)
/e/(a^2-b^2)/(b+(-a^2+b^2)^(1/2))/((-a^2+b^2)^(1/2)-b)/cos(d*x+c)/(e*sin(d*
x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] e^(-3/2)*integrate(1/((b*cos(d*x + c) + a)*sin(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\sin\left(c+dx\right)\right)^{\frac{3}{2}}\left(a+b\cos\left(c+dx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(3/2),x)
[Out] Integral(1/((e*sin(c + d*x))**(3/2)*(a + b*cos(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(e^(-3/2)/((b*cos(d*x + c) + a)*sin(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))),x)
[Out] int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))), x)
```

3.66 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=447

$$\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{7/4}de^{5/2}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{7/4}de^{5/2}}+\frac{2(b-a\cos(c+dx))}{3(a^2-b^2)de(e\sin(c+dx))^{3/2}}+$$

```
[Out] b^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2
+b^2)^(7/4)/d/e^(5/2)+b^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/(-a^2+b^2)^(7/4)/d/e^(5/2)+2/3*(b-a*cos(d*x+c))/(a^2-b^2)
/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^
(1/2)/(a^2-b^2)/d/e^2/(e*sin(d*x+c))^(1/2)+a*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*
b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b*(b-
(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+a*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(
b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b*(b+(-a
^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2775, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{5/2}(b^2-a^2)^{7/4}}+\frac{2a\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)}{3de^2(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{ab^2\sqrt{\sin(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)}{de^2(a^2-b^2)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}-\frac{ab^2\sqrt{\sin(c+dx)}\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)}{de^2(a^2-b^2)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}+\frac{2(b-a\cos(c+dx))}{3de(a^2-b^2)(e\sin(c+dx))^{3/2}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{5/2}(b^2-a^2)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/((-a^2 + b^2)^(7/4)*d*e^(5/2)) + (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c
+ d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(7/4)*d*e^(5/2)) + (2
*(b - a*Cos[c + d*x]))/(3*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + (2*a*El
lipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*(a^2 - b^2)*d*e^2*Sqr
t[e*Sin[c + d*x]]) - (a*b^2*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - P
i/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 +
 b^2]))*d*e^2*Sqrt[e*Sin[c + d*x]]) - (a*b^2*EllipticPi[(2*b)/(b + Sqrt[-a^
2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b
*(b + Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx = \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a^2}{2}+\frac{3b^2}{2}-\frac{1}{2}ab\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+}}}{3\left(a^2-b^2\right)e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3\left(a^2-b^2\right)e^2} - \cdots b$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} - \frac{(ab^2)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{\cdots}\right.}}{2\left(-a^2+b^2\right.}$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\mathrm{si}}}{3\left(a^2-b^2\right)de^2\sqrt{e\sin(c}}$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\mathrm{si}}}{3\left(a^2-b^2\right)de^2\sqrt{e\sin(c}}$$

$$= \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{7/4}de^{5/2}} + \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin}}{\sqrt[4]{-a^2+}}\right.}{\left(-a^2+b^2\right)^{7/4}de}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 31.77, size = 1192, normalized size = 2.67

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*(-b + a*Cos[c + d*x])*Sin[c + d*x])/(3*(a^2 - b^2)*d*(e*Sin[c + d*x])^(
5/2)) + (Sin[c + d*x]^(5/2)*((2*a*b*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c +
d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2
)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1
/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c +
 d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^
2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b
^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2
])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c
```

```
+ d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]
^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1,
9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a
^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^
2)) + (2*(a^2 - 3*b^2)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/
8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 +
 b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^
2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[
Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*
(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3
/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[
c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)
/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*S
in[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[
c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*
(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2]))
/(3*(a - b)*(a + b)*d*(e*Sin[c + d*x])^(5/2))
```

Maple [A]
time = 0.26, size = 820, normalized size = 1.83

method	result
default	$\frac{b^{3}\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{e\sin(dx+c)+\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin\left(dx+c\right)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}{e\sin(dx+c)-\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{e\sin\left(dx+c\right)}\,\sqrt{2}+\sqrt{\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}}}\right)}{4e(a-b)(a+b)\left(a^{2}e^{2}-b^{2}e^{2}\right)}+\frac{b^{3}\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{b^{2}}\right)^{\frac{1}{4}}\sqrt{2}\,\mathrm{arc}}{2e(a-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*b^3/e/(a-b)/(a+b)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*
ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^
2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+
c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+1/2*b^3/e/(a-b)/(a+b)*(e^2*(a
^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/
b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+1/2*b^3/e/(a-b)/(a+b)*(e^2*(a^2-b^2)/b^2
```

```
)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*
(e*sin(d*x+c))^(1/2)-1)+2/3*b/e/(a^2-b^2)/(e*sin(d*x+c))^(3/2)+(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)*a/e^2*(1/3/(a^2-b^2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2
)/(cos(d*x+c)^2-1)*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+
c))-1/(a-b)/(a+b)*b^2*(-1/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^
2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1
/2*2^(1/2))+1/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)
/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))
)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] e^(-5/2)*integrate(1/((b*cos(d*x + c) + a)*sin(d*x + c)^(5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\sin\left(c+dx\right)\right)^{\frac{5}{2}}\left(a+b\cos\left(c+dx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(5/2),x)
[Out] Integral(1/((e*sin(c + d*x))**(5/2)*(a + b*cos(c + d*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(e^(-5/2)/((b*cos(d*x + c) + a)*sin(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\sin\left(c+d\,x\right)\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))),x)
[Out] int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))), x)
```

3.67 $\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=501

$$-\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{9/4}de^{7/2}}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{(-a^2+b^2)^{9/4}de^{7/2}}+\frac{2(b-a\cos(c+dx))}{5(a^2-b^2)de(e\sin(c+dx))^{5/2}}$$

```
[Out] -b^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^
2+b^2)^(9/4)/d/e^(7/2)+b^(7/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(9/4)/d/e^(7/2)+2/5*(b-a*cos(d*x+c))/(a^2-b^2
)/d/e/(e*sin(d*x+c))^(5/2)-2/5*(5*b^3+a*(3*a^2-8*b^2)*cos(d*x+c))/(a^2-b^2)
^2/d/e^3/(e*sin(d*x+c))^(1/2)-a*b^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^
2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(b-(-a^2+b^2)^(1/2))/
(e*sin(d*x+c))^(1/2)-a*b^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/
4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2))
,2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(b+(-a^2+b^2)^(1/2))/(e*sin(d*
x+c))^(1/2)+2/5*a*(3*a^2-8*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+
c))^(1/2)/(a^2-b^2)^2/d/e^4/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.87, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2775, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{7/2}(b^2-a^2)^{9/4}}-\frac{2a(3a^2-8b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4(a^2-b^2)^2\sqrt{\sin(c+dx)}}+\frac{2(b-a\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{de^{7/2}(b^2-a^2)^{9/4}}-\frac{2(a(3a^2-8b^2)\cos(c+dx)+5b^3)}{5de^3(a^2-b^2)^2\sqrt{e\sin(c+dx)}}+\frac{ab^3\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{de^3(a^2-b^2)^2\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{ab^3\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{de^3(a^2-b^2)^2\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] -((b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/((-a^2 + b^2)^(9/4)*d*e^(7/2))) + (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin
[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/((-a^2 + b^2)^(9/4)*d*e^(7/2)) +
 (2*(b - a*Cos[c + d*x]))/(5*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(5/2)) - (2*(
5*b^3 + a*(3*a^2 - 8*b^2)*Cos[c + d*x]))/(5*(a^2 - b^2)^2*d*e^3*Sqrt[e*Sin[
c + d*x]]) + (a*b^3*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*e^3*Sq
rt[e*Sin[c + d*x]]) + (a*b^3*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(b + Sqrt[-a^2 + b^2])
*d*e^3*Sqrt[e*Sin[c + d*x]]) - (2*a*(3*a^2 - 8*b^2)*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*(a^2 - b^2)^2*d*e^4*Sqrt[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx = \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+\frac{5b^2}{2}-\frac{3}{2}ab\cos(c+dx)}{(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}}\,c}{5\left(a^2-b^2\right)e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} - \frac{2(5b^3+a(3a^2-8b^2)\cos(c+}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+d}}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} - \frac{2(5b^3+a(3a^2-8b^2)\cos(c+}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+d}}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} - \frac{2(5b^3+a(3a^2-8b^2)\cos(c+}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+d}}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} - \frac{2(5b^3+a(3a^2-8b^2)\cos(c+}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+d}}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} - \frac{2(5b^3+a(3a^2-8b^2)\cos(c+}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+d}}$$

$$= -\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{\left(-a^2+b^2\right)^{9/4}de^{7/2}} + \frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin}}{\sqrt[4]{-a^2+}}\right.}{\left(-a^2+b^2\right)^{9/4}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 26.57, size = 881, normalized size = 1.76

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] (((-2*(5*b^3 + 3*a^3*Cos[c + d*x] - 8*a*b^2*Cos[c + d*x])*Csc[c + d*x])/(5*
(a^2 - b^2)^2) - (2*(-b + a*Cos[c + d*x])*Csc[c + d*x]^3)/(5*(a^2 - b^2)))*
Sin[c + d*x]^4)/(d*(e*Sin[c + d*x])^(7/2)) - (Sin[c + d*x]^(7/2)*(((3*a^3*b
 - 8*a*b^3)*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sq
rt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2
]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sq
rt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[
```

```
Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*
Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2
*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c +
d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2
)) + (2*(3*a^4 - 8*a^2*b^2 - 5*b^4)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1
- ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (
(1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b
^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d
*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c
 + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3
/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c +
 d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos
[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(5*(a - b)^2*(a + b)^2*d*(e*Sin[c +
d*x])^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(535) = 1070$.
time = 0.24, size = 1080, normalized size = 2.16

method	result	size
default	Expression too large to display	1080

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*b^3/e^3/(a-b)^2/(a+b)^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln((e*sin(d
*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)
/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2
^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-1/2*b^3/e^3/(a-b)^2/(a+b)^2/(e^2*(a^2-b^
2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c
))^(1/2)+1)-1/2*b^3/e^3/(a-b)^2/(a+b)^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+2/5*b/e/(a+
b)/(a-b)/(e*sin(d*x+c))^(5/2)-2*b^3/e^3/(a-b)^2/(a+b)^2/(e*sin(d*x+c))^(1/2
)+1/10*a/e^3*(5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/
2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2))*(-
a^2+b^2)^(1/2)*b^3+5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(7/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2
))*b^4-5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*Elli
pticPi((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)
^(1/2)*b^3+5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*
EllipticPi((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^4-12
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-
sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^4+32*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2
*b^2+6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*Ellipt
```

```
icF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^4-16*(-sin(d*x+c)+1)^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2
))*a^2*b^2-12*a^4*cos(d*x+c)^4*sin(d*x+c)+32*a^2*b^2*cos(d*x+c)^4*sin(d*x+c
)+16*a^4*cos(d*x+c)^2*sin(d*x+c)-36*a^2*b^2*cos(d*x+c)^2*sin(d*x+c))/sin(d*
x+c)^3/(a^2-b^2)^2/(b+(-a^2+b^2)^(1/2))/((-a^2+b^2)^(1/2)-b)/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sin(d*x + c))/(b*cos(d*x + c)^5*e^(7/2) + a*cos(d*x + c)^4*e^
(7/2) - 2*b*cos(d*x + c)^3*e^(7/2) - 2*a*cos(d*x + c)^2*e^(7/2) + b*cos(d*x
 + c)*e^(7/2) + a*e^(7/2)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-7/2)/((b*cos(d*x + c) + a)*sin(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\sin\left(c+d\,x\right)\right)^{7/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))),x)
[Out] int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))), x)
```

3.68 $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=557

$$\frac{9a(-a^2+b^2)^{5/4}\,e^{11/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{11/2}d}+\frac{9a(-a^2+b^2)^{5/4}\,e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{11/2}d} -$$

```
[Out] 9/2*a*(-a^2+b^2)^(5/4)*e^(11/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/b^(11/2)/d+9/2*a*(-a^2+b^2)^(5/4)*e^(11/2)*arctanh(b^(1/
2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/d-9/35*e^3*(7*a-
5*b*cos(d*x+c))*(e*sin(d*x+c))^(5/2)/b^3/d+e*(e*sin(d*x+c))^(9/2)/b/d/(a+b*
cos(d*x+c))+3/7*(21*a^4-28*a^2*b^2+5*b^4)*e^6*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))*sin(d*x+c)^(1/2)/b^6/d/(e*sin(d*x+c))^(1/2)-9/2*a^2*(a^2-b^2)^2*e^6*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/
2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^6/
d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-9/2*a^2*(a^2-b^2)^2*e^6
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(c
os(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)
/b^6/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+3/7*e^5*(21*a*(a^2
-b^2)-b*(7*a^2-5*b^2)*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^5/d
```

Rubi [A]
time = 1.03, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{9ae^{11/2}(b^2-a^2)^{5/4}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2b^{11/2}d}+\frac{9ae^{11/2}(b^2-a^2)^{5/4}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2b^{11/2}d}+\frac{9a^2e^6(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2b^6d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}+\frac{9a^2e^6(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2b^6d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}+\frac{3e^5\sqrt{e\sin(c+dx)}\,(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))}{7b^5d}-\frac{3e^6(21a^4-28a^2b^2+5b^4)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{7b^6d\sqrt{e\sin(c+dx)}}-\frac{9e^3(e\sin(c+dx))^{5/2}(7a-5b\cos(c+dx))}{35b^3d}+\frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (9*a*(-a^2 + b^2)^(5/4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a
^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(11/2)*d) + (9*a*(-a^2 + b^2)^(5/4)*e^(11/2
)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*
b^(11/2)*d) - (3*(21*a^4 - 28*a^2*b^2 + 5*b^4)*e^6*EllipticF[(c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(7*b^6*d*Sqrt[e*Sin[c + d*x]]) + (9*a^2*(a^2 -
 b^2)^2*e^6*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]
*Sqrt[Sin[c + d*x]])/(2*b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c
 + d*x]]) + (9*a^2*(a^2 - b^2)^2*e^6*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]
), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^6*(a^2 - b*(b + Sqrt[-a^
2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (3*e^5*(21*a*(a^2 - b^2) - b*(7*a^2 -
```

```
5*b^2)*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(7*b^5*d) - (9*e^3*(7*a - 5*b*Co
s[c + d*x])*(e*Sin[c + d*x])^(5/2))/(35*b^3*d) + (e*(e*Sin[c + d*x])^(9/2))
/(b*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} - \frac{(9e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{a+b\cos(c+dx)}\,dx}{2b}$$

$$= -\frac{9e^3(7a-5b\cos(c+dx))(e\sin(c+dx))^{5/2}}{35b^3d} + \frac{e(e\sin(c+dx))^{9/2}}{bd(a+b\cos(c+dx))} - \frac{(9e^4)\int$$

$$= \frac{3e^5(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{7b^5d} - \frac{9e^3(7a-5b\,\mathrm{c}$$

$$= \frac{3e^5(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{7b^5d} - \frac{9e^3(7a-5b\,\mathrm{c}$$

$$= \frac{3e^5(21a(a^2-b^2)-b(7a^2-5b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{7b^5d} - \frac{9e^3(7a-5b\,\mathrm{c}$$

$$= -\frac{3(21a^4-28a^2b^2+5b^4)\,e^6F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{7b^6d\sqrt{e\sin(c+dx)}} + \frac{3e^5(21a(a^2$$

$$= -\frac{3(21a^4-28a^2b^2+5b^4)\,e^6F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{7b^6d\sqrt{e\sin(c+dx)}} + \frac{9a^2(-a^2+$$

$$= \frac{9a(-a^2+b^2)^{5/4}\,e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{11/2}d} + \frac{9a(-a^2+b^2)^{5/4}\,e^{11/2}\,\mathrm{ta}}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 45.17, size = 2029, normalized size = 3.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((((-28*a^2 + 17*b^2)*Cos[c + d*x])/(14*b^4) + (-a^2 + b^2)^2/(b^5*(a + b*C
os[c + d*x])) + (2*a*Cos[2*(c + d*x)])/(5*b^3) - Cos[3*(c + d*x)]/(14*b^2))
*Csc[c + d*x]^5*(e*Sin[c + d*x])^(11/2))/d - ((e*Sin[c + d*x])^(11/2)*((2*(
35*a^4 - 126*a^2*b^2 + 75*b^4)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^
2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/
```

```
4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)]
 - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]
] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/
4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3
/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Si
n[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((
-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x
]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (
b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4,
Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 +
b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) +
 (2*(70*a^3*b - 93*a*b^3)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((
-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^
2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 +
 b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sq
rt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + b^2)
^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*S
in[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*
(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]
^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^
2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, S
in[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b
^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2]
) + ((-140*a^3*b + 147*a*b^3)*Cos[c + d*x]*Cos[2*(c + d*x)]*(a + b*Sqrt[1 -
 Sin[c + d*x]^2])*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*
Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1
/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-
a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + ((1/4 - I/4)*(-2*a^2 + b^
2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d
*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a
^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Sin
[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c
+ d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 -
b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*Appell
F1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2
*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a
^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*S
in[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]
^2)))))/((a + b*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x]^
2])))/(70*b^5*d*Sin[c + d*x]^(11/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2509 vs.

2(579) = 1158.
time = 0.46, size = 2510, normalized size = 4.51

method	result	size
default	Expression too large to display	2510

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-4/5*e^3*a/b^3*(e*sin(d*x+c))^(5/2)+8*e^5*a^3/b^5*(e*sin(d*x+c))^(1/2)-8*e
^5*a/b^3*(e*sin(d*x+c))^(1/2)+e^7*a^5/b^5*(e*sin(d*x+c))^(1/2)/(-b^2*cos(d*
x+c)^2*e^2+a^2*e^2)-2*e^7*a^3/b^3*(e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2*e
^2+a^2*e^2)+e^7*a/b*(e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)-9/
4*e^7*a^5/b^5*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^
(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+9/2*e^7*a^3/b^3*(e^
2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b
^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)-9/4*e^7*a/b*(e^2*(a^2-b^2)/b^2)^(1/4
)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin
(d*x+c))^(1/2)+1)-9/4*e^7*a^5/b^5*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^
2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)
+9/2*e^7*a^3/b^3*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan
(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)-9/4*e^7*a/b*(e^2
*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)-9/8*e^7*a^5/b^5*(e^2*(a^2-b^2)/b^2)^(
1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(
e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(
a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))
)+9/4*e^7*a^3/b^3*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e
*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^
2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(
1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-9/8*e^7*a/b*(e^2*(a^2-b^2)/b^2)^(1
/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e
*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a
^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))
+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^6*(1/7/b^6/(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)*(-2*b^4*cos(d*x+c)^4*sin(d*x+c)+35*a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2
))-49*a^2*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+11*b^4*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2
*2^(1/2))-14*a^2*b^2*cos(d*x+c)^2*sin(d*x+c)+10*b^4*cos(d*x+c)^2*sin(d*x+c)
)-(-7*a^6+15*a^4*b^2-9*a^2*b^4+b^6)/b^6*(-1/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x
+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a
```

```
^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(
1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2
)/b),1/2*2^(1/2)))-2*a^2*(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/b^6*(1/2*b^2/e/a^2/(
a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)+1/4/a^2/
(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))
-5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/
b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1
/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2
)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))
+5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/
b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1
/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2
)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))
))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] e^(11/2)*integrate(sin(d*x + c)^(11/2)/(b*cos(d*x + c) + a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(11/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\sin\left(c+d\,x\right))^{11/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^2, x)
```

3.69 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=473

$$-\frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{9/2}d}+\frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{9/2}d}-$$

```
[Out] -7/2*a*(-a^2+b^2)^(3/4)*e^(9/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))/b^(9/2)/d+7/2*a*(-a^2+b^2)^(3/4)*e^(9/2)*arctanh(b^(1/2)
*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/d-7/15*e^3*(5*a-3*b
*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/b^3/d+e*(e*sin(d*x+c))^(7/2)/b/d/(a+b*cos
(d*x+c))+7/2*a^2*(a^2-b^2)*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1
/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^5/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1
/2)+7/2*a^2*(a^2-b^2)*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),
2^(1/2))*sin(d*x+c)^(1/2)/b^5/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-7
/5*(5*a^2-3*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b
^4/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2944, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{7ae^{9/2}(b^2-a^2)^{3/4}\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}+\frac{7ae^{9/2}(b^2-a^2)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{9/2}d}-\frac{7a^2e^5(a^2-b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2b^5d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{7a^2e^5(a^2-b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2b^5d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}+\frac{7e^4(5a^2-3b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}}-\frac{7e^3(e\sin(c+dx))^{3/2}(5a-3b\cos(c+dx))}{15b^3d}+\frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-7*a*(-a^2 + b^2)^(3/4)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a
^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(9/2)*d) + (7*a*(-a^2 + b^2)^(3/4)*e^(9/2)*
ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^
(9/2)*d) - (7*a^2*(a^2 - b^2)*e^5*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^5*(b - Sqrt[-a^2 + b^2])*d*
Sqrt[e*Sin[c + d*x]]) - (7*a^2*(a^2 - b^2)*e^5*EllipticPi[(2*b)/(b + Sqrt[-
a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^5*(b + Sqrt[-a
^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (7*(5*a^2 - 3*b^2)*e^4*EllipticE[(c -
Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*b^4*d*Sqrt[Sin[c + d*x]]) - (7*e
^3*(5*a - 3*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*b^3*d) + (e*(e*Sin[
c + d*x])^(7/2))/(b*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} - \frac{(7e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{a+b\cos(c+dx)}\,dx}{2b}$$

$$= -\frac{7e^3(5a-3b\cos(c+dx))(e\sin(c+dx))^{3/2}}{15b^3d} + \frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} - \frac{(7e^4)\int}{}$$

$$= -\frac{7e^3(5a-3b\cos(c+dx))(e\sin(c+dx))^{3/2}}{15b^3d} + \frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} + \frac{(7(5a^2}{}$$

$$= -\frac{7e^3(5a-3b\cos(c+dx))(e\sin(c+dx))^{3/2}}{15b^3d} + \frac{e(e\sin(c+dx))^{7/2}}{bd(a+b\cos(c+dx))} + \frac{(7a^2(a}{}$$

$$= \frac{7(5a^2-3b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5b^4d\sqrt{\sin(c+dx)}} - \frac{7e^3(5a-3b\cos(c+d}{15b^3}$$

$$= -\frac{7a^2(a^2-b^2)\,e^5\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{2b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{7a^2(a}{}$$

$$= -\frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{9/2}d} + \frac{7a(-a^2+b^2)^{3/4}\,e^{9/2}\tan}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 34.73, size = 835, normalized size = 1.77

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (7*(e*Sin[c + d*x])^(9/2)*(((5*a^2 - 3*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^
2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^
2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(
1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c
+ d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^
2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1
/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]
```

```
^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*C
os[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*A
rcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcT
an[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[
-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*S
in[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sq
rt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*Ap
pellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]
*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a
 + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(10*b^3*d*Sin[c + d*x]^(9/2)
) + (Csc[c + d*x]^4*(e*Sin[c + d*x])^(9/2)*((-4*a*Sin[c + d*x])/(3*b^3) + (
-(a^2*Sin[c + d*x]) + b^2*Sin[c + d*x])/(b^3*(a + b*Cos[c + d*x])) + Sin[2*
(c + d*x)]/(5*b^2)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1994 vs. $2(499) = 998$.
time = 0.42, size = 1995, normalized size = 4.22

method	result	size
default	Expression too large to display	1995

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-4/3*e^3*a/b^3*(e*sin(d*x+c))^(3/2)-e^5*a^3/b^3*(e*sin(d*x+c))^(3/2)/(-b^2
*cos(d*x+c)^2*e^2+a^2*e^2)+e^5*a/b*(e*sin(d*x+c))^(3/2)/(-b^2*cos(d*x+c)^2*
e^2+a^2*e^2)+7/8*e^5*a^3/b^5/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln((e*sin(d*
x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/
b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^
(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+7/4*e^5*a^3/b^5/(e^2*(a^2-b^2)/b^2)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+7
/4*e^5*a^3/b^5/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b
^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)-7/8*e^5*a/b^3/(e^2*(a^2-b^2)/b^2)^(1
/4)*2^(1/2)*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)
*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)
*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-7/4*e^5*a/b^3/(e^
2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*
sin(d*x+c))^(1/2)+1)-7/4*e^5*a/b^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)*e^5*(-1/5/b^4/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(30*(-sin
(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*
x+c)+1)^(1/2),1/2*2^(1/2))*a^2-16*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-15*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin
(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+8*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
```

```
(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2+2*b
^2*cos(d*x+c)^4-2*cos(d*x+c)^2*b^2)-(5*a^4-6*a^2*b^2+b^4)/b^4*(-1/2/b^2*(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1
/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+
b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2
*2^(1/2)))+2*a^2*(a^4-2*a^2*b^2+b^4)/b^4*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c
)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-1/2/a^2/(a^2-b^
2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c
)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+1/4/a^
2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2
))-3/8/(a^2-b^2)/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi
((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b
^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^
(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+
b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+
b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2
*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] e^(9/2)*integrate(sin(d*x + c)^(9/2)/(b*cos(d*x + c) + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(9/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\sin\left(c+d\,x\right))^{9/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^2, x)
```

3.70 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=487

$$\frac{5a\sqrt[4]{-a^2+b^2}\,e^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{7/2}d}+\frac{5a\sqrt[4]{-a^2+b^2}\,e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2b^{7/2}d}+\frac{5(3}{}$$

```
[Out] 5/2*a*(-a^2+b^2)^(1/4)*e^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^
2)^(1/4)/e^(1/2))/b^(7/2)/d+5/2*a*(-a^2+b^2)^(1/4)*e^(7/2)*arctanh(b^(1/2)*
(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/d+e*(e*sin(d*x+c))^(
5/2)/b/d/(a+b*cos(d*x+c))-5/3*(3*a^2-b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))*sin(d*x+c)^(1/2)/b^4/d/(e*sin(d*x+c))^(1/2)+5/2*a^2*(a^2-b^2)*e^4*(sin(1
/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*
c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/
(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+5/2*a^2*(a^2-b^2)*e^4*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1
/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^4
/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-5/3*e^3*(3*a-b*cos(d*x
+c))*(e*sin(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.74, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d}+\frac{5ae^{7/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2b^{7/2}d}+\frac{5e^4(3a^2-b^2)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3b^4d\sqrt{e\sin(c+dx)}}-\frac{5a^2e^4(a^2-b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2b^4d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}-\frac{5a^2e^4(a^2-b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2b^4d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}-\frac{5e^3\sqrt{e\sin(c+dx)}\,(3a-b\cos(c+dx))}{3b^3d}+\frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (5*a*(-a^2 + b^2)^(1/4)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(7/2)*d) + (5*a*(-a^2 + b^2)^(1/4)*e^(7/2)*A
rcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(
7/2)*d) + (5*(3*a^2 - b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(3*b^4*d*Sqrt[e*Sin[c + d*x]]) - (5*a^2*(a^2 - b^2)*e^4*EllipticPi
[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(
2*b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (5*a^2*(a^
2 - b^2)*e^4*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2
]*Sqrt[Sin[c + d*x]])/(2*b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[
c + d*x]]) - (5*e^3*(3*a - b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*b^3*d)
+ (e*(e*Sin[c + d*x])^(5/2))/(b*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} - \frac{(5e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{a+b\cos(c+dx)}\,dx}{2b}$$

$$= -\frac{5e^3(3a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} + \frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} - \frac{(5e^4)\int -}{(a+}$$

$$= -\frac{5e^3(3a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} + \frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} - \frac{(5a(a^2-b}{}$$

$$= -\frac{5e^3(3a-b\cos(c+dx))\sqrt{e\sin(c+dx)}}{3b^3d} + \frac{e(e\sin(c+dx))^{5/2}}{bd(a+b\cos(c+dx))} - \frac{\left(5a^2\sqrt{-a}\right.}{}$$

$$= \frac{5(3a^2-b^2)\,e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}} - \frac{5e^3(3a-b\cos(c+dx))\sqrt{}}{3b^3d}$$

$$= \frac{5(3a^2-b^2)\,e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{3b^4d\sqrt{e\sin(c+dx)}} + \frac{5a^2\sqrt{-a^2+b^2}\,e^4\Pi\left(\frac{}{b-\sqrt{-}}\right.}{2b^4\left(b-\sqrt{}\right.}$$

$$= \frac{5a\sqrt[4]{-a^2+b^2}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{7/2}d} + \frac{5a\sqrt[4]{-a^2+b^2}\,e^{7/2}\tanh^{-1}\left(\right.}{2b^{7/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 44.56, size = 1956, normalized size = 4.02

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (((2*Cos[c + d*x])/(3*b^2) + (-a^2 + b^2)/(b^3*(a + b*Cos[c + d*x])))*Csc[c
 + d*x]^3*(e*Sin[c + d*x])^(7/2))/d + ((e*Sin[c + d*x])^(7/2)*((2*(3*a^2 -
5*b^2)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (
Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt
[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Lo
g[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*
```

```
AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^
2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[
1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(
2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^
2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin
[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2
)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (8*a*b*Cos[c + d*x]*(a
+ b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)
*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sq
rt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log
[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/
2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c +
d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4,
Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4,
 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 +
b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c
 + d*x])*Sqrt[1 - Sin[c + d*x]^2]) - (6*a*b*Cos[c + d*x]*Cos[2*(c + d*x)]*(
a + b*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1
 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2
)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin
[c + d*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + ((1/4 - I/4
)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*
Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/
4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)
^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4))
 + (4*Sqrt[Sin[c + d*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^
2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) +
 (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c +
d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a
^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c
 + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c +
d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1
+ Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 -
 Sin[c + d*x]^2])))/(6*b^3*d*Sin[c + d*x]^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1963 vs. $2(513) = 1026$.

time = 0.37, size = 1964, normalized size = 4.03

method	result	size
default	Expression too large to display	1964

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] (-4*e^3*a/b^3*(e*sin(d*x+c))^(1/2)-e^5*a^3/b^3*(e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)+e^5*a/b*(e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)+5/8*e^5*a^3/b^3*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-5/8*e^5*a/b*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+5/4*e^5*a^3/b^3*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)-5/4*e^5*a/b*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+5/4*e^5*a^3/b^3*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)-5/4*e^5*a/b*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^4*(-1/3/b^4/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(9*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-2*b^2*cos(d*x+c)^2*sin(d*x+c))-1/b^4*(5*a^4-6*a^2*b^2+b^4)*(-1/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+2*a^2*(a^4-2*a^2*b^2+b^4)/b^4*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)

```
+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2
+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] e^(7/2)*integrate(sin(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*cos(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,\sin\left(c+d\,x\right)\right)}^{7/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^2, x)
```

3.71 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=404

$$-\frac{3ae^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}\,d}+\frac{3ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}\,d}+\frac{3a^2e^3\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\right.}{2b^3\left(b-\sqrt{-a^2\right.}}$$

```
[Out] -3/2*a*e^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/b^(5/2)/(-a^2+b^2)^(1/4)/d+3/2*a*e^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(
1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(1/4)/d+e*(e*sin(d*x+c))^
(3/2)/b/d/(a+b*cos(d*x+c))-3/2*a^2*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2
+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^3/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*
x+c))^(1/2)-3/2*a^2*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^
(1/2))*sin(d*x+c)^(1/2)/b^3/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+3*e
^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(
cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^2/d/sin(d*x+c)^(1
/2)
```

Rubi [A]

time = 0.53, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2772, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{3ae^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}}+\frac{3ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{5/2}d\sqrt[4]{b^2-a^2}}+\frac{3a^2e^3\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2b^3d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{3a^2e^3\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2b^3d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}+\frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))}-\frac{3e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-3*a*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*b^(5/2)*(-a^2 + b^2)^(1/4)*d) + (3*a*e^(5/2)*ArcTanh[(Sqrt[b]*Sq
rt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(5/2)*(-a^2 + b^2)^
(1/4)*d) + (3*a^2*e^3*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^3*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c
 + d*x]]) + (3*a^2*e^3*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^3*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[
c + d*x]]) - (3*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/
(b^2*d*Sqrt[Sin[c + d*x]]) + (e*(e*Sin[c + d*x])^(3/2))/(b*d*(a + b*Cos[c +
 d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))} - \frac{(3e^2)\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{2b}$$

$$= \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))} - \frac{(3e^2)\int \sqrt{e\sin(c+dx)}\,dx}{2b^2} + \frac{(3ae^2)\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}}{2b^2}$$

$$= \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))} - \frac{(3a^2e^3)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b^3} +$$

$$= -\frac{3e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{b^2d\sqrt{\sin(c+dx)}} + \frac{e(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))} - \frac{(3ae^3)\,\mathrm{Su}}{}$$

$$= \frac{3a^2e^3\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{2b^3\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{3a^2e^3\Pi\left(\frac{2b}{b+\sqrt{-a^2}}\right.}{2b^3\left(b+\right.}$$

$$= -\frac{3ae^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}\,d} + \frac{3ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{5/2}\sqrt[4]{-a^2+b^2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 80.52, size = 366, normalized size = 0.91

$$\frac{(e\sin(c+dx))^{5/2}\left(8b^{3/2}\csc(c+dx)+\frac{\left(a+b\sqrt{\cos^2(c+dx)}\right)\left(3\sqrt{2}\,a(a^2-b^2)^{3/4}\left(2\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-2\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\sin(c+dx)}+b\sin(c+dx)\right)+\log\left(\sqrt{a^2-b^2}+\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\sin(c+dx)}+b\sin(c+dx)\right)\right)+8b^{5/2}F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\sin^2(c+dx),\frac{b^2\sin^2(c+dx)}{-a^2+b^2}\right)\sin^{\frac{3}{2}}(c+dx)\right)}{(a^2-b^2)\sin^{\frac{5}{2}}(c+dx)}\right)}{8b^{5/2}d(a+b\cos(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((e*Sin[c + d*x])^(5/2)*(8*b^(3/2)*Csc[c + d*x] + ((a + b*Sqrt[Cos[c + d*x]
^2])*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c +
 d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(
5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
 + b^2)]*Sin[c + d*x]^(3/2)))/((a^2 - b^2)*Sin[c + d*x]^(5/2))))/(8*b^(5/2)
*d*(a + b*Cos[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1721 vs. $2(436) = 872$.

time = 0.25, size = 1722, normalized size = 4.26

method	result	size
default	Expression too large to display	1722

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (e^3*a/b*(e*sin(d*x+c))^(3/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)-3/8*e^3*a/b^3
/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e
^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1
/2)))-3/4*e^3*a/b^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(
a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)-3/4*e^3*a/b^3/(e^2*(a^2-b^2)/b^
2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1
/2)-1)-1/4*e^3*a^2*(12*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(5/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^3-6*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(
1/2),1/2*2^(1/2))*b^3-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(5/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(
1/2))*(-a^2+b^2)^(1/2)*b^2-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(5/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/
2*2^(1/2))*b^3+3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5
/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-
a^2+b^2)^(1/2)*b^2-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(5/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2)
)*b^3+12*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elli
pticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2*b-12*(-sin(d*x+c)+1)^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^
(1/2))*b^3-6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*
EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2*b+6*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2
*2^(1/2))*b^3-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2))*(-
a^2+b^2)^(1/2)*a^2+3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2
))*(-a^2+b^2)^(1/2)*b^2-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2
^(1/2))*a^2*b+3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2))*b^
3+3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticP
i((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^(1/2
```

```
)*a^2-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellip
ticPi((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*(-a^2+b^2)^
(1/2)*b^2-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*E
llipticPi((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2*b+3
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((
-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3+4*b^3*sin(d*x+
c)^4-4*sin(d*x+c)^2*b^3)/b^3/(b+(-a^2+b^2)^(1/2))/((-a^2+b^2)^(1/2)-b)/(-co
s(d*x+c)^2*b^2+a^2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] e^(5/2)*integrate(sin(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\sin(c+d\,x))^{5/2}}{(a+b\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2, x)
```

3.72 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=418

$$\frac{ae^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d}+\frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d}-\frac{e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c}}{b^2d\sqrt{e\sin(c+dx)}}$$

```
[Out] 1/2*a*e^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))
/b^(3/2)/(-a^2+b^2)^(3/4)/d+1/2*a*e^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1
/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(3/4)/d+e^2*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+
1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/b^2/d/(e*sin(d*x+c))^(1/2)-1/2*a^2*e^2*(
sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos
(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b
^2/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-1/2*a^2*e^2*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+
1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/d/(a
^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+e*(e*sin(d*x+c))^(1/2)/b/d/
(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.58, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2772, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{ae^{3/2}\operatorname{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}d\left(b^2-a^2\right)^{3/4}}+\frac{a^2e^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2b^2d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}+\frac{a^2e^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2b^2d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}+\frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2b^{3/2}d\left(b^2-a^2\right)^{3/4}}+\frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))}-\frac{e^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{b^2d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (a*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(2*b^(3/2)*(-a^2 + b^2)^(3/4)*d) + (a*e^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[e*
Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*b^(3/2)*(-a^2 + b^2)^(3/4)
*d) - (e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(b^2*d*Sqrt
[e*Sin[c + d*x]]) + (a^2*e^2*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]
))*d*Sqrt[e*Sin[c + d*x]]) + (a^2*e^2*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2
]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b^2*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (e*Sqrt[e*Sin[c + d*x]])/(b*d*(a + b*
Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx = \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))} - \frac{e^2\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{2b}$$

$$= \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))} - \frac{e^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{2b^2} + \frac{(ae^2)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\sin}}}{2b^2}$$

$$= \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))} - \frac{(a^2e^2)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+dx)\right)}\,dx}{4b^2\sqrt{-a^2+b^2}} -$$

$$= -\frac{e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{b^2d\sqrt{e\sin(c+dx)}} + \frac{e\sqrt{e\sin(c+dx)}}{bd(a+b\cos(c+dx))} - \frac{(ae^3)\,\mathrm{Subs}}{}$$

$$= -\frac{e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{b^2d\sqrt{e\sin(c+dx)}} + \frac{a^2e^2\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+\right.\right.}{2b^2\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)}$$

$$= \frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} + \frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2b^{3/2}\left(-a^2+b^2\right)^{3/4}d} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 19.53, size = 614, normalized size = 1.47

$$\frac{\csc(c+dx)(e\sin(c+dx))^{3/2}}{bd(a+b\cos(c+dx))} - \frac{\cos^2(c+dx)(e\sin(c+dx))^{3/2}\left(a+b\sqrt{1-\sin^2(c+dx)}\right)\left(\frac{a\left(-2\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\sin(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\sin(c+dx)}+b\sin(c+dx)\right)+\log\left(\sqrt{a^2-b^2}+\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\sin(c+dx)}+b\sin(c+dx)\right)\right)}{4\sqrt{2}\sqrt{b}\left(a^2-b^2\right)^{3/4}} + \frac{5b(a^2-b^2)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\sin^2(c+dx),\frac{b^2\sin^2(c+dx)}{-a^2+b^2}\right)\sqrt{\sin(c+dx)}\sqrt{1-\sin^2(c+dx)}}{\left(-5(a^2-b^2)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\sin^2(c+dx),\frac{b^2\sin^2(c+dx)}{-a^2+b^2}\right)+2\left(2b^2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\sin^2(c+dx),\frac{b^2\sin^2(c+dx)}{-a^2+b^2}\right)+(a^2-b^2)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\sin^2(c+dx),\frac{b^2\sin^2(c+dx)}{-a^2+b^2}\right)\right)\sin^2(c+dx)\right)\left(a^2+b^2(-1+\sin^2(c+dx))\right)}\right)}{bd(a+b\cos(c+dx))\sin^{\frac{3}{2}}(c+dx)\left(1-\sin^2(c+dx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]*(e*Sin[c + d*x])^(3/2))/(b*d*(a + b*Cos[c + d*x])) - (Cos[c +
 d*x]^2*(e*Sin[c + d*x])^(3/2)*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*Arc
Tan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[
1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2
- b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d
*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c +
 d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^
2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(
-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*
AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^
2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x
```

```
]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2,
 (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c
 + d*x]^2)))))/(b*d*(a + b*Cos[c + d*x])*Sin[c + d*x]^(3/2)*(1 - Sin[c + d*
x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1459 vs. $2(449) = 898$.
time = 0.36, size = 1460, normalized size = 3.49

method	result	size
default	Expression too large to display	1460

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (e^3*a/b*(e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)-1/8*e^3*a/b*(
e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a
^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/
(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(
a^2-b^2)/b^2)^(1/2)))-1/4*e^3*a/b*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^
2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)
-1/4*e^3*a/b*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(
1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)*e^2*(1/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/
2),1/2*2^(1/2))-(-3*a^2+b^2)/b^2*(-1/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)
^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2
+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/
2*2^(1/2)))-2*a^2*(a^2-b^2)/b^2*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*si
n(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/b/(-a^2+b^2)
^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+
1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*b/(-a^2+b^
2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+5/8/(a^2-b^2)/b/(-a^2+b^2)
^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+
1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/4/a^2/(a^2-b^2)*b/(-a^2+b^
2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
```

```
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] e^(3/2)*integrate(sin(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
[Out] integral(e^(3/2)*sin(d*x + c)^(3/2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
[Out] integrate(e^(3/2)*sin(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e \sin (c+d x))^{3/2}}{(a+b \cos (c+d x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2, x)
```

3.73 $\int \frac{\sqrt{e \sin(c+dx)}}{(a+b\cos(c+dx))^2}\, dx$

Optimal. Leaf size=438

$$\frac{a\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\sqrt{b}\,(-a^2+b^2)^{5/4}\,d}-\frac{a\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\sqrt{b}\,(-a^2+b^2)^{5/4}\,d}+\frac{a^2e\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(c-\right.}{2b\,(a^2-b^2)\left(b-\sqrt{-a^2}\right.}$$

```
[Out] -b*(e*sin(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*cos(d*x+c))+1/2*a*arctan(b^(1/2)
*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(5/4)/d/
b^(1/2)-1/2*a*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)*e^(1/2)/(-a^2+b^2)^(5/4)/d/b^(1/2)-1/2*a^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2
)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/
(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b-(-a^2+b^2)^
(1/2))/(e*sin(d*x+c))^(1/2)-1/2*a^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+
b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b+(-a^2+b^2)^(1/2))/(e
*sin(d*x+c))^(1/2)-(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2
*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^
2-b^2)/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2773, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{a\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}\,d\,(b^2-a^2)^{5/4}}-\frac{a\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2\sqrt{b}\,d\,(b^2-a^2)^{5/4}}-\frac{b(e\sin(c+dx))^{3/2}}{de\,(a^2-b^2)\,(a+b\cos(c+dx))}+\frac{E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\,(a^2-b^2)\,\sqrt{\sin(c+dx)}}+\frac{a^2e\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2bd\,(a^2-b^2)\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{a^2e\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2bd\,(a^2-b^2)\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (a*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(2*Sqrt[b]*(-a^2 + b^2)^(5/4)*d) - (a*Sqrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*
Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*Sqrt[b]*(-a^2 + b^2)^(5/4)
*d) + (a^2*e*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2
]*Sqrt[Sin[c + d*x]])/(2*b*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[
c + d*x]]) + (a^2*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*b*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt
[e*Sin[c + d*x]]) + (EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])
/((a^2 - b^2)*d*Sqrt[Sin[c + d*x]]) - (b*(e*Sin[c + d*x])^(3/2))/((a^2 - b^
2)*d*e*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = -\frac{b(e\sin(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\cos(c+dx))} + \frac{\int \frac{\left(-a-\frac{1}{2}b\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}\,dx}{-a^2+b^2}$$

$$= -\frac{b(e\sin(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\cos(c+dx))} + \frac{\int \sqrt{e\sin(c+dx)}\,dx}{2(a^2-b^2)} + \frac{a\int \frac{\sqrt{e\sin(c+dx)}}{a+b\cos(c+dx)}}{2(a^2-b^2)}$$

$$= -\frac{b(e\sin(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\cos(c+dx))} - \frac{(a^2e)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{-a^2+b^2}-b\sin(c+\right.}}{4b(a^2-b^2)}$$

$$= \frac{E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{\sin(c+dx)}} - \frac{b(e\sin(c+dx))^{3/2}}{(a^2-b^2)\,de(a+b\cos(c+dx))} - \frac{(abe}{}$$

$$= \frac{a^2e\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{2b(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{a^2e\Pi\left(\frac{2b}{b+\sqrt{-a^2+b^2}}\right.}{2b(a^2-b^2)\left(b+\right.}$$

$$= \frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\left(-a^2+b^2\right)^{5/4}d} - \frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\sqrt{b}\left(-a^2+b^2\right)^{5/4}d} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 34.07, size = 786, normalized size = 1.79

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (b*Sin[c + d*x]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)*d*(a + b*Cos[c + d*x]))
 + (Sqrt[e*Sin[c + d*x]]*((Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2
*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*Arc
Tan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[
a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c
 + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin
[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c
 + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sq
rt[1 - Sin[c + d*x]^2]))/(12*Sqrt[b]*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 -
 Sin[c + d*x]^2)) + (4*a*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*
```

```
Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqr
t[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 +
I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[
Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1,
 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)
)/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*
Sqrt[1 - Sin[c + d*x]^2])))/(2*(a - b)*(a + b)*d*Sqrt[Sin[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1397 vs. $2(471) = 942$.

time = 0.31, size = 1398, normalized size = 3.19

method	result	size
default	Expression too large to display	1398

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-e^3*a*b*(e*sin(d*x+c))^(3/2)/(a^2*e^2-b^2*e^2)/(-b^2*cos(d*x+c)^2*e^2+a^2
*e^2)-1/8*e^3*a/b/(a^2*e^2-b^2*e^2)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln((e
*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^
2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(
1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-1/4*e^3*a/b/(a^2*e^2-b^2*e^2)/(e^2
*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*s
in(d*x+c))^(1/2)+1)-1/4*e^3*a/b/(a^2*e^2-b^2*e^2)/(e^2*(a^2-b^2)/b^2)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e*(1/2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b
^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*
2^(1/2))+1/2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1
/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-s
in(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+2*a^2*(1/2*b^2/e/a
^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^
2+a^2)-1/2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/
2),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^
2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2
^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*Ellip
ticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/8/(a^2-
b^2)/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
```

```
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(
-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
[Out] e^(1/2)*integrate(sqrt(sin(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
[Out] integral(e^(1/2)*sqrt(sin(d*x + c))/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \sin{\left(c + dx \right)}}}{\left(a + b \cos{\left(c + dx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*cos(d*x+c))**2,x)
[Out] Integral(sqrt(e*sin(c + d*x))/(a + b*cos(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(e^(1/2)*sqrt(sin(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,\sin\left(c+d\,x\right)}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2, x)
```

3.74 $\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=445

$$-\frac{3a\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{7/4}d\sqrt{e}}-\frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{7/4}d\sqrt{e}}-\frac{F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\mathrm{si}}}{(a^2-b^2)\,d\sqrt{e\sin(c+}}$$

```
[Out] -3/2*a*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2
)/(-a^2+b^2)^(7/4)/d/e^(1/2)-3/2*a*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a
^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(7/4)/d/e^(1/2)+(sin(1/2*c+1/4*Pi
+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2
*d*x),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/(e*sin(d*x+c))^(1/2)-3/2*a^2*(s
in(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(
1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a
^2-b^2)/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-3/2*a^2*(sin(1/
2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c
+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^
2)/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-b*(e*sin(d*x+c))^(1/
2)/(a^2-b^2)/d/e/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.58, antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2773, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{3a\sqrt{b}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\,\left(b^2-a^2\right)^{7/4}}-\frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2d\sqrt{e}\,\left(b^2-a^2\right)^{7/4}}-\frac{b\sqrt{e\sin(c+dx)}}{de\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{3a^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2d\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}+\frac{3a^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2d\left(a^2-b^2\right)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (-3*a*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*(-a^2 + b^2)^(7/4)*d*Sqrt[e]) - (3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(7/4)*d*
Sqrt[e]) - (EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^
2)*d*Sqrt[e*Sin[c + d*x]]) + (3*a^2*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2])
, (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)*(a^2 - b*(b - S
qrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (3*a^2*EllipticPi[(2*b)/(b + Sq
rt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)*
(a^2 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (b*Sqrt[e*Sin[c
+ d*x]])/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = -\frac{b\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de(a+b\cos(c+dx))} + \frac{\int \frac{-a+\frac{1}{2}b\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{e\sin(c+}}}{-a^2+b^2}$$

$$= -\frac{b\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de(a+b\cos(c+dx))} - \frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{2(a^2-b^2)} + \cdots$$

$$= -\frac{b\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de(a+b\cos(c+dx))} + \frac{(3a^2)\int \frac{}{\sqrt{e\sin(c+dx)}\,(\sqrt{}}}{4(-a^2}$$

$$= -\frac{F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{e\sin(c+dx)}} - \frac{b\sqrt{e\sin(c+d}}{(a^2-b^2)\,de(a+b\cos}$$

$$= -\frac{F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\,d\sqrt{e\sin(c+dx)}} - \frac{3a^2\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}}\right.}{2(-a^2+b^2)^{3/2}\,(b}$$

$$= -\frac{3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2(-a^2+b^2)^{7/4}\,d\sqrt{e}} - \frac{3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{}}\right.}{2(-a^2+b^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 30.69, size = 1182, normalized size = 2.66

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] -((b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]
)) + (Sqrt[Sin[c + d*x]]*((-2*b*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]
^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1
/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)]
 - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x
]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*S
in[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/(
(-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*
```

```
x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2,
(b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4,
 Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 +
 b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2))
+ (4*a*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]
*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2
*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[
Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/
4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2
- b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^
2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*Appe
llF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] -
 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(
-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2
*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*
x]^2)))))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(2*(a - b)*(a +
 b)*d*Sqrt[e*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1368 vs. $2(476) = 952$.
time = 0.33, size = 1369, normalized size = 3.08

method	result	size
default	Expression too large to display	1369

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*b*e^3*(e*sin(d*x+c))^(1/2)/(a^2*e^2-b^2*e^2)/(-b^2*cos(d*x+c)^2*e^2+a^2
*e^2)-3/8*a*b*e^3/(a^2*e^2-b^2*e^2)^2*(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln(
(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(
a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))
^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-3/4*a*b*e^3/(a^2*e^2-b^2*e^2)^2*
(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*
(e*sin(d*x+c))^(1/2)+1)-3/4*a*b*e^3/(a^2*e^2-b^2*e^2)^2*(e^2*(a^2-b^2)/b^2)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2
)-1)+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(1/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)
+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2
+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+
(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/
b),1/2*2^(1/2))+2*a^2*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^
(1/2)/(-cos(d*x+c)^2*b^2+a^2)+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*si
```

```
n(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Ellipt
icF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1
/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(
-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*
e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2)
,1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1
/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(
-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*
e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2)
,1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate(1/((b*cos(d*x + c) + a)^2*sqrt(sin(d*x + c))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(1/((b^2*cos(d*x + c)^2*e^(1/2) + 2*a*b*cos(d*x + c)*e^(1/2) + a^2*
e^(1/2))*sqrt(sin(d*x + c))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\sin{(c+dx)}}\,(a+b\cos{(c+dx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*sin(c + d*x))*(a + b*cos(c + d*x))**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(e^(-1/2)/((b*cos(d*x + c) + a)^2*sqrt(sin(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,\sin\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2),x)
[Out] int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2), x)
```

3.75 $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=507

$$\frac{5ab^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}}-\frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}}-\frac{b}{\left(a^2-b^2\right)de(a+b\cos(c+}$$

```
[Out] 5/2*a*b^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))
/(-a^2+b^2)^(9/4)/d/e^(3/2)-5/2*a*b^(3/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1
/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(9/4)/d/e^(3/2)-b/(a^2-b^2)/d/e/(a
+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2)+(5*a*b-(2*a^2+3*b^2)*cos(d*x+c))/(a^2-b
^2)^2/d/e/(e*sin(d*x+c))^(1/2)+5/2*a^2*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a
^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e/(b-(-a^2+b^2)^(1/2
))/(e*sin(d*x+c))^(1/2)+5/2*a^2*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)
^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e/(b+(-a^2+b^2)^(1/2))/(e*s
in(d*x+c))^(1/2)+(2*a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c
))^(1/2)/(a^2-b^2)^2/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.81, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{5ab^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2de^{3/2}\left(b^2-a^2\right)^{9/4}}-\frac{\left(2a^2+3b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\left(a^2-b^2\right)^2\sqrt{\sin(c+dx)}}+\frac{5ab-\left(2a^2+3b^2\right)\cos(c+dx)}{de\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}}-\frac{b}{de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}\,(a+b\cos(c+dx))}-\frac{5a^2b\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2de\left(a^2-b^2\right)^2\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{5a^2b\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2de\left(a^2-b^2\right)^2\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}-\frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2de^{3/2}\left(b^2-a^2\right)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (5*a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[e])])/(2*(-a^2 + b^2)^(9/4)*d*e^(3/2)) - (5*a*b^(3/2)*ArcTanh[(Sqrt[b]*Sqr
t[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(9/4)*d*e
^(3/2)) - b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]) + (
5*a*b - (2*a^2 + 3*b^2)*Cos[c + d*x])/((a^2 - b^2)^2*d*e*Sqrt[e*Sin[c + d*x
]]) - (5*a^2*b*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2,
 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*e*Sqrt[e*
Sin[c + d*x]]) - (5*a^2*b*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)^2*(b + Sqrt[-a^2 + b^2])*
d*e*Sqrt[e*Sin[c + d*x]]) - ((2*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2,
2]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^2*d*e^2*Sqrt[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{\int \frac{-a\cdots}{(a+b\cos(c\cdots}}{\cdots}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5ab-(2a^2\cdots}{(a^2-b^2)^2\cdots}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5ab-(2a^2\cdots}{(a^2-b^2)^2\cdots}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5ab-(2a^2\cdots}{(a^2-b^2)^2\cdots}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5ab-(2a^2\cdots}{(a^2-b^2)^2\cdots}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))\sqrt{e\sin(c+dx)}} + \frac{5ab-(2a^2\cdots}{(a^2-b^2)^2\cdots}$$

$$= \frac{5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{9/4}de^{3/2}} - \frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\cdots}}{\sqrt[4]{-\cdots}}\right)}{2\left(-a^2+b^2\right)^{9/\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 26.48, size = 865, normalized size = 1.71

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sin[c + d*x]^2*((-2*(-2*a*b + a^2*Cos[c + d*x] + b^2*Cos[c + d*x])*Csc[c +
 d*x])/(a^2 - b^2)^2 + (b^3*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x
]))))/(d*(e*Sin[c + d*x])^(3/2)) - (Sin[c + d*x]^(3/2)*(((2*a^2*b + 3*b^3)*
Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b
]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sq
rt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b
```

```
^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]
]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x
]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(1
2*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(2*a
^3 + 8*a*b^2)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqr
t[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[S
in[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*
(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 +
 b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c +
 d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c
 + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 -
 b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Si
n[c + d*x]^2])))/(2*(a - b)^2*(a + b)^2*d*(e*Sin[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2080 vs. $2(536) = 1072$.
time = 0.30, size = 2081, normalized size = 4.10

method	result	size
default	Expression too large to display	2081

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/e*a*b^3/(a-b)^2/(a+b)^2*(e*sin(d*x+c))^(3/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*
e^2)+5/8/e*a*b/(a-b)^2/(a+b)^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln((e*sin(
d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*
2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+5/4/e*a*b/(a-b)^2/(a+b)^2/(e^2*(a^2-b^2
)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c)
)^(1/2)+1)+5/4/e*a*b/(a-b)^2/(a+b)^2/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+4/e*a*b/(a^2-b
^2)^2/(e*sin(d*x+c))^(1/2)-1/4/e*a^2*(5*(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2
),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2))*a^2*b-5*(-a^2+b^2)^(1/2)*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((-sin(d*x+c)+
1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2*b+5*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-b
/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2))*b^4+5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2
)^(1/2)),1/2*2^(1/2))*b^4+12*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(5/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^4-6*(-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c
)+1)^(1/2),1/2*2^(1/2))*b^4-5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1
```

```
/2*2^(1/2))*b^4-5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b
^4+8*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elliptic
E((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^4-12*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))
*b^4-2*a^2*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+5*(-a^2+b^2)^(1/2)*(-sin(d*x
+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticPi((-sin(d*x+c
)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2))*b^3-5*(-a^2+b^2)^(1/2)*(-si
n(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticPi((-sin(
d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3+8*(-sin(d*x+c)+1)^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticE((-sin(d*x+c)+1)^(1/2
),1/2*2^(1/2))*a^2*b^2-4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2*b^2-5*(-a^2+b^
2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elli
pticPi((-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2))*b^3+5*(-a
^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
*EllipticPi((-sin(d*x+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*b^3+5
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((
-sin(d*x+c)+1)^(1/2),-b/((-a^2+b^2)^(1/2)-b),1/2*2^(1/2))*a^2*b^2+5*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((-sin(d*x
+c)+1)^(1/2),b/(b+(-a^2+b^2)^(1/2)),1/2*2^(1/2))*a^2*b^2+4*(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/
2),1/2*2^(1/2))*a^2*b^2+8*a^2*b^2*cos(d*x+c)^4-8*a^2*b^2*cos(d*x+c)^2+12*b^
4*cos(d*x+c)^4-8*a^4*cos(d*x+c)^2-4*b^4*cos(d*x+c)^2-4*a^4*(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/
2),1/2*2^(1/2))+6*b^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)))/(-cos(d*x+c)^2*b^2+a
^2)/(a^2-b^2)^2/(b+(-a^2+b^2)^(1/2))/((-a^2+b^2)^(1/2)-b)/cos(d*x+c)/(e*sin
(d*x+c))^(1/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(e^(-3/2)/((b*cos(d*x + c) + a)^2*sin(d*x + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\sin\left(c+d\,x\right)\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2),x)
[Out] int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2), x)
```

3.76 $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=530

$$-\frac{7ab^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}}-\frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{11/4}de^{5/2}}-\frac{b}{\left(a^2-b^2\right)de(a+b\cos(c+}$$

```
[Out] -7/2*a*b^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2)
)/(-a^2+b^2)^(11/4)/d/e^(5/2)-7/2*a*b^(5/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^
(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(11/4)/d/e^(5/2)-b/(a^2-b^2)/d/e
/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2)+1/3*(7*a*b-(2*a^2+5*b^2)*cos(d*x+c))
/(a^2-b^2)^2/d/e/(e*sin(d*x+c))^(3/2)-1/3*(2*a^2+5*b^2)*(sin(1/2*c+1/4*Pi+1
/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d
*x),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(e*sin(d*x+c))^(1/2)+7/2*a^
2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/(a^2-b^2)^2/d/e^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+7
/2*a^2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*
x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1
/2)
```

Rubi [A]
time = 0.90, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2945, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{7ab^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{5/2}\left(b^2-a^2\right)^{11/4}}+\frac{\left(2a^2+5b^2\right)\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)}{3de^2\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}}-\frac{7a^2b^2\sqrt{\sin(c+dx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)}{2de^2\left(a^2-b^2\right)^2\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}-\frac{7a^2b^2\sqrt{\sin(c+dx)}\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)}{2de^2\left(a^2-b^2\right)^2\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}+\frac{7ab-\left(2a^2+5b^2\right)\cos(c+dx)}{3de\left(a^2-b^2\right)^2(e\sin(c+dx))^{3/2}}-\frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{2de^{5/2}\left(b^2-a^2\right)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-7*a*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqr
t[e])])/(2*(-a^2 + b^2)^(11/4)*d*e^(5/2)) - (7*a*b^(5/2)*ArcTanh[(Sqrt[b]*S
qrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(11/4)*
d*e^(5/2)) - b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)
) + (7*a*b - (2*a^2 + 5*b^2)*Cos[c + d*x])/(3*(a^2 - b^2)^2*d*e*(e*Sin[c +
d*x])^(3/2)) + ((2*a^2 + 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
 + d*x]])/(3*(a^2 - b^2)^2*d*e^2*Sqrt[e*Sin[c + d*x]]) - (7*a^2*b^2*Ellipti
cPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(2*(a^2 - b^2)^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Sin[c + d*
x]]) - (7*a^2*b^2*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)
```

```
/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)^2*(a^2 - b*(b + Sqrt[-a^2 + b^2])
)*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{\int \frac{}{(a+b\cos}}{}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{7ab - (}{3\,(a^2 -}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{7ab - (}{3\,(a^2 -}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{7ab - (}{3\,(a^2 -}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{7ab - (}{3\,(a^2 -}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{3/2}} + \frac{7ab - (}{3\,(a^2 -}$$

$$= -\frac{7ab^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\,(-a^2+b^2)^{11/4}\,de^{5/2}} - \frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{2\,(-a^2+b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 33.71, size = 1257, normalized size = 2.37

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] ((b^3/((a^2 - b^2)^2*(a + b*Cos[c + d*x])) - (2*(-2*a*b + a^2*Cos[c + d*x]
+ b^2*Cos[c + d*x])*Csc[c + d*x]^2)/(3*(a^2 - b^2)^2))*Sin[c + d*x]^3)/(d*(
e*Sin[c + d*x])^(5/2)) + (Sin[c + d*x]^(5/2)*((2*(2*a^2*b + 5*b^3)*Cos[c +
d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]
*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqr
t[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]
*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^
```

```
2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]
))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -
1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c
+ d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5
/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[
5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^
2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-
a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2))))/((a + b*C
os[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(2*a^3 - 16*a*b^2)*Cos[c + d*x]*(a
+ b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)
*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sq
rt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log
[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/
2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c +
d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4,
Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4,
 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 +
b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c
 + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(6*(a - b)^2*(a + b)^2*d*(e*Sin[c + d*
x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1598 vs. $2(556) = 1112$.
time = 0.45, size = 1599, normalized size = 3.02

method	result	size
default	Expression too large to display	1599

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/e*a*b^3/(a-b)^2/(a+b)^2*(e*sin(d*x+c))^(1/2)/(-b^2*cos(d*x+c)^2*e^2+a^2*
e^2)+7/8/e*a*b^3/(a-b)^2/(a+b)^2*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2
)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2
^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(
e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+7/4/e*a*b^3/(a-b)^2
/(a+b)^2*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)
/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+7/4/e*a*b^3/(a-b)^2/(a+b
)^2*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2
*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+4/3/e*a*b/(a^2-b^2)^2/(e*sin(
d*x+c))^(3/2)-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e^2*(1/3*(-a^2-b^2)/(a^2-b^
2)^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*((-sin(d*x+c)+1)^(1
```

```
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2)
,1/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+c))+b^2*(a^2+b^2)/(a-b)^2/(a+b)^2*(-1/
2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi
((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^
2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+2*a^2*b^2/(a-b)/(a+b)*(1/
2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+
a^2)+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2)
,1/2*2^(1/2))-5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^
2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1
/2*2^(1/2))+1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-
a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b)
,1/2*2^(1/2))+5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^
2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1
/2*2^(1/2))-1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-
a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b)
,1/2*2^(1/2)))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-5/2)/((b*cos(d*x + c) + a)^2*sin(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\sin\left(c+d\,x\right)\right)^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2), x)
```

3.77 $\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=590

$$\frac{9ab^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{13/4}de^{7/2}}-\frac{9ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{13/4}de^{7/2}}-\frac{b}{(a^2-b^2)\,de(a+b\cos(c+d}$$

[Out] 9/2*a*b^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/e^(7/2)-9/2*a*b^(7/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/e^(7/2)-b/(a^2-b^2)/d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2)+1/5*(9*a*b-(2*a^2+7*b^2)*cos(d*x+c))/(a^2-b^2)^2/d/e/(e*sin(d*x+c))^(5/2)-3/5*(15*a*b^3+(2*a^4-10*a^2*b^2-7*b^4)*cos(d*x+c))/(a^2-b^2)^3/d/e^3/(e*sin(d*x+c))^(1/2)-9/2*a^2*b^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3/d/e^3/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-9/2*a^2*b^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3/d/e^3/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+3/5*(2*a^4-10*a^2*b^2-7*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^3/d/e^4/sin(d*x+c)^(1/2)

Rubi [A]
time = 1.08, antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{9ab^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2de^{7/2}\left(b^2-a^2\right)^{13/4}}-\frac{b}{de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}+\frac{9ab-\left(2a^2+7b^2\right)\cos(c+dx)}{5de\left(a^2-b^2\right)^2(e\sin(c+dx))^{5/2}}-\frac{9ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{2de^{7/2}\left(b^2-a^2\right)^{13/4}}+\frac{9a^2b^3\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2de^3\left(a^2-b^2\right)^3\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{9a^2b^3\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2de^3\left(a^2-b^2\right)^3\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}-\frac{3\left(2a^4-10a^2b^2-7b^4\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\left(a^2-b^2\right)^3\sqrt{\sin(c+dx)}}-\frac{3\left(\left(2a^4-10a^2b^2-7b^4\right)\cos(c+dx)+15ab^3\right)}{5de^3\left(a^2-b^2\right)^3\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] (9*a*b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(13/4)*d*e^(7/2)) - (9*a*b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(2*(-a^2 + b^2)^(13/4)*d*e^(7/2)) - b/((a^2 - b^2)*d*e*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2)) + (9*a*b - (2*a^2 + 7*b^2)*Cos[c + d*x])/(5*(a^2 - b^2)^2*d*e*(e*Sin[c + d*x])^(5/2)) - (3*(15*a*b^3 + (2*a^4 - 10*a^2*b^2 - 7*b^4)*Cos[c + d*x]))/(5*(a^2 - b^2)^3*d*e^3*Sqrt[e*Sin[c + d*x]]) + (9*a^2*b^3*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Sin[c + d*x]]) + (9*a^2*b^3*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*
```

```
x]])/(2*(a^2 - b^2)^3*(b + Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Sin[c + d*x]]) -
(3*(2*a^4 - 10*a^2*b^2 - 7*b^4)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin
[c + d*x]])/(5*(a^2 - b^2)^3*d*e^4*Sqrt[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^2(e\sin(c+dx))^{7/2}}\,dx = -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{\int \frac{-}{(a+b\cos(}}{}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9ab-(2}{5\,(a^2-b^2}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9ab-(2}{5\,(a^2-b^2}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9ab-(2}{5\,(a^2-b^2}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9ab-(2}{5\,(a^2-b^2}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9ab-(2}{5\,(a^2-b^2}$$

$$= -\frac{b}{(a^2-b^2)\,de(a+b\cos(c+dx))(e\sin(c+dx))^{5/2}} + \frac{9ab-(2}{5\,(a^2-b^2}$$

$$= \frac{9ab^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{2\left(-a^2+b^2\right)^{13/4}de^{7/2}} - \frac{9ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{}}{\sqrt[4]{-}}\right.}{2\left(-a^2+b^2\right)^{13}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 26.60, size = 950, normalized size = 1.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] (Sin[c + d*x]^4*((-2*(20*a*b^3 + 3*a^4*Cos[c + d*x] - 15*a^2*b^2*Cos[c + d*
x] - 8*b^4*Cos[c + d*x])*Csc[c + d*x])/(5*(a^2 - b^2)^3) - (2*(-2*a*b + a^2
*Cos[c + d*x] + b^2*Cos[c + d*x])*Csc[c + d*x]^3)/(5*(a^2 - b^2)^2) - (b^5*
Sin[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x]))))/(d*(e*Sin[c + d*x])^(7
```

```
/2)) - (3*Sin[c + d*x]^(7/2)*(((2*a^4*b - 10*a^2*b^3 - 7*b^5)*Cos[c + d*x]^
2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c
+ d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*
x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]
*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2
)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 +
b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a
^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(2*a^5 - 10*a^3*b
^2 - 22*a*b^4)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sq
rt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]
*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2
+ b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c
+ d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[
c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2
- b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - S
in[c + d*x]^2])))/(10*(a - b)^3*(a + b)^3*d*(e*Sin[c + d*x])^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1836 vs. $2(612) = 1224$.
time = 0.49, size = 1837, normalized size = 3.11

method	result	size
default	Expression too large to display	1837

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/e^3*a*b^5/(a-b)^3/(a+b)^3*(e*sin(d*x+c))^(3/2)/(-b^2*cos(d*x+c)^2*e^2+a
^2*e^2)-9/8/e^3*a*b^3/(a-b)^3/(a+b)^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln(
(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(
a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))
^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-9/4/e^3*a*b^3/(a-b)^3/(a+b)^3/(e
^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e
*sin(d*x+c))^(1/2)+1)-9/4/e^3*a*b^3/(a-b)^3/(a+b)^3/(e^2*(a^2-b^2)/b^2)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)
+4/5/e*a*b/(a+b)^2/(a-b)^2/(e*sin(d*x+c))^(5/2)-8/e^3*a*b^3/(a-b)^3/(a+b)^3
/(e*sin(d*x+c))^(1/2)-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e^3*(-b^4*(3*a^2+b^
2)/(a-b)^3/(a+b)^3*(-1/2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*El
lipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^
2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)
^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1
/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))-1/5*(-a^2-b^2)/(a^2-b^2)^2/(cos(
```

```
d*x+c)^2*e*sin(d*x+c))^(1/2)/sin(d*x+c)/(cos(d*x+c)^2-1)*(6*(-sin(d*x+c)+1)
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1
/2),1/2*2^(1/2))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+6*cos(d*x+c)^4*sin(d*x+c
)-8*cos(d*x+c)^2*sin(d*x+c))-2*a^2*b^4/(a-b)^2/(a+b)^2*(1/2*b^2/e/a^2/(a^2-
b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-1
/2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1
/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2
^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1
/2),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)
/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+
1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-
sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/
2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2
)^(1/2)/b),1/2*2^(1/2)))+b^2*(3*a^2+b^2)/(a^2-b^2)^3*(2*(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))-(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*
EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2)/(cos(d*x+c)^2*
e*sin(d*x+c))^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**2/(e*sin(d*x+c))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(e^(-7/2)/((b*cos(d*x + c) + a)^2*sin(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\sin\left(c+d\,x\right)\right)^{7/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2),x)
[Out] int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2), x)
```

3.78 $\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=590

$$\frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}\,d}-\frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin}}{\sqrt[4]{-a^2}}\right.}{8b^{13/2}\sqrt[4]{-a^2+b^2}\,d}$$

```
[Out] 11/8*(9*a^4-11*a^2*b^2+2*b^4)*e^(13/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/
(-a^2+b^2)^(1/4)/e^(1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d-11/8*(9*a^4-11*a^2*b^
2+2*b^4)*e^(13/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(
1/2))/b^(13/2)/(-a^2+b^2)^(1/4)/d-11/60*e^5*(45*a^2-10*b^2-27*a*b*cos(d*x+c
))*(e*sin(d*x+c))^(3/2)/b^5/d+11/28*e^3*(9*a+2*b*cos(d*x+c))*(e*sin(d*x+c))
^(7/2)/b^3/d/(a+b*cos(d*x+c))+1/2*e*(e*sin(d*x+c))^(11/2)/b/d/(a+b*cos(d*x+
c))^2+11/8*a*(9*a^4-11*a^2*b^2+2*b^4)*e^7*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-
a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^7/d/(b-(-a^2+b^2)^(1/2))/(e*sin
(d*x+c))^(1/2)+11/8*a*(9*a^4-11*a^2*b^2+2*b^4)*e^7*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),
2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^7/d/(b+(-a^2+b^2)^(1/2
))/(e*sin(d*x+c))^(1/2)-11/20*a*(45*a^2-37*b^2)*e^6*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),
2^(1/2))*(e*sin(d*x+c))^(1/2)/b^6/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2772, 2942, 2944, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{11ae^6(45a^2-37b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20b^6d\sqrt{\sin(c+dx)}}-\frac{11e^5(e\sin(c+dx))^{3/2}(5(9a^2-2b^2)-27ab\cos(c+dx))}{60b^5d}+\frac{11e^{13/2}(9a^4-11a^2b^2+2b^4)\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{13/2}d\sqrt[4]{b^2-a^2}}-\frac{11e^{13/2}(9a^4-11a^2b^2+2b^4)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8b^{13/2}d\sqrt[4]{b^2-a^2}}-\frac{11ae^7(9a^4-11a^2b^2+2b^4)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8b^7d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{11ae^7(9a^4-11a^2b^2+2b^4)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8b^7d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}+\frac{11e^3(e\sin(c+dx))^{7/2}(9a+2b\cos(c+dx))}{28b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(13/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (11*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^(13/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*
x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(13/2)*(-a^2 + b^2)^(1/4)*d) - (11
*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^(13/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]
])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(13/2)*(-a^2 + b^2)^(1/4)*d) - (11*a
*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^7*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*b^7*(b - Sqrt[-a^2 + b^2])*d*
Sqrt[e*Sin[c + d*x]]) - (11*a*(9*a^4 - 11*a^2*b^2 + 2*b^4)*e^7*EllipticPi[(
2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*
b^7*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (11*a*(45*a^2 - 37*b^2
```

```
)*e^6*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(20*b^6*d*Sqrt
[Sin[c + d*x]]) - (11*e^5*(5*(9*a^2 - 2*b^2) - 27*a*b*Cos[c + d*x])*(e*Sin[
c + d*x])^(3/2))/(60*b^5*d) + (11*e^3*(9*a + 2*b*Cos[c + d*x])*(e*Sin[c + d
*x])^(7/2))/(28*b^3*d*(a + b*Cos[c + d*x])) + (e*(e*Sin[c + d*x])^(11/2))/(
2*b*d*(a + b*Cos[c + d*x])^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
```

```
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
```

```
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{13/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(11e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^2}\,dx}{4b}$$

$$= \frac{11e^3(9a+2b\cos(c+dx))(e\sin(c+dx))^{7/2}}{28b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{11/2}}{2bd(a+b\cos(c+dx))^2} + \frac{(11e^4}{}$$

$$= -\frac{11e^5(5(9a^2-2b^2)-27ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{60b^5d} + \frac{11e^3(9a+2b\cos(}{28b^3d(a}$$

$$= -\frac{11e^5(5(9a^2-2b^2)-27ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{60b^5d} + \frac{11e^3(9a+2b\cos(}{28b^3d(a}$$

$$= -\frac{11e^5(5(9a^2-2b^2)-27ab\cos(c+dx))\,(e\sin(c+dx))^{3/2}}{60b^5d} + \frac{11e^3(9a+2b\cos(}{28b^3d(a}$$

$$= \frac{11a(45a^2-37b^2)\,e^6E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20b^6d\sqrt{\sin(c+dx)}} - \frac{11e^5(5(9a^2-2b^2}{}$$

$$= -\frac{11a(9a^4-11a^2b^2+2b^4)\,e^7\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+d}}{8b^7\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}}$$

$$= \frac{11(9a^4-11a^2b^2+2b^4)\,e^{13/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{13/2}\sqrt[4]{-a^2+b^2}\,d} - \frac{11(9a^4-11a^2b^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 35.16, size = 930, normalized size = 1.58

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(13/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (11*(e*Sin[c + d*x])^(13/2)*(((45*a^3 - 37*a*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]
*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^
2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 -
b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[
Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^
2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3
/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c
+ d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a
 + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(18*a^2*b - 10*b^3)*Cos[c + d
*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2
+ b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b
^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt
[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]
*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2
 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c +
 d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 -
 Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(40*b^
5*d*Sin[c + d*x]^(13/2)) + (Csc[c + d*x]^6*(e*Sin[c + d*x])^(13/2)*(((-168*
a^2 + 65*b^2)*Sin[c + d*x])/(42*b^5) - (19*(a^3*Sin[c + d*x] - a*b^2*Sin[c
+ d*x]))/(4*b^5*(a + b*Cos[c + d*x])) + (a^4*Sin[c + d*x] - 2*a^2*b^2*Sin[c
 + d*x] + b^4*Sin[c + d*x])/(2*b^5*(a + b*Cos[c + d*x])^2) + (3*a*Sin[2*(c
+ d*x)])/(5*b^4) - Sin[3*(c + d*x)]/(14*b^3)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3820 vs. 2(607) = 1214.
time = 0.76, size = 3821, normalized size = 6.48

method	result	size
default	Expression too large to display	3821

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2/7*e^3/b^3*(e*sin(d*x+c))^(7/2)-4*e^5/b^5*(e*sin(d*x+c))^(3/2)*a^2+4/3*e^
5/b^3*(e*sin(d*x+c))^(3/2)-21/4*e^7/b^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(
```

e*sin(d*x+c))^(7/2)*a^4+23/4*e^7/b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin (d*x+c))^(7/2)*a^2-1/2*e^7*b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(7/2)-17/4*e^9/b^5/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(3/2)*a^6+9*e^9/b^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(3/2)*a^4- 21/4*e^9/b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(3/2)*a^2+1/2*e ^9*b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(3/2)+99/32*e^7/b^7/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*a^4*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+ (e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^ (1/2)))+99/16*e^7/b^7/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*a^4*arctan(2^(1/2)/ (e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+99/16*e^7/b^7/(e^2*(a^2-b ^2)/b^2)^(1/4)*2^(1/2)*a^4*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)-121/32*e^7/b^5/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*a^2*ln((e *sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^ 2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-121/16*e^7/b^5/(e^2*(a^2-b^2)/b^2) ^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^ (1/2)+1)-121/16*e^7/b^5/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+11/16*e^7/b^3/(e^2*(a^2 -b^2)/b^2)^(1/4)*2^(1/2)*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^ 2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+11/8 *e^7/b^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^ 2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+11/8*e^7/b^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^7*a*(1/5/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/ 2)*(100*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellip ticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-78*(-sin(d*x+c)+1)^(1/2)*(2*sin (d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/ 2))*b^2-50*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*El lipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+39*(-sin(d*x+c)+1)^(1/2)*(2* sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^ (1/2))*b^2+6*b^2*cos(d*x+c)^4-6*cos(d*x+c)^2*b^2)+3*(7*a^4-10*a^2*b^2+3*b^4)/b^6*(-1/2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/ 2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-si n(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+ c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^ 2+b^2)^(1/2)/b),1/2*2^(1/2)))-3*(5*a^6-11*a^4*b^2+7*a^2*b^4-b^6)/b^6*(1/2*b ^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-1/2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2) *sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+ 1)^(1/2),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c) +2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-si n(d*x+c)+1)^(1/2),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(-sin(d*x+c)+1)^(1/2)*(2*s

```
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-
a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b)
,1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)
*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/8
/(a^2-b^2)/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin
(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),
1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+4*a^2*(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/
b^6*(1/4*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(
-cos(d*x+c)^2*b^2+a^2)^2+1/16*b^2*(11*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*sin(d*x+
c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-11/16/a^2/(a^2
-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3
/8/a^4/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2
*2^(1/2))*b^2+11/32/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(13/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(13/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,\sin\left(c+d\,x\right)\right)^{13/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(13/2)/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((e*sin(c + d*x))^(13/2)/(a + b*cos(c + d*x))^3, x)
```

3.79 $\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=604

$$-\frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{11/2}\left(-a^2+b^2\right)^{3/4}d}-\frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(}{\sqrt[4]{-a^2+b}}\right.}{8b^{11/2}\left(-a^2+b^2\right)^{3/4}d}$$

```
[Out] -9/8*(7*a^4-9*a^2*b^2+2*b^4)*e^(11/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(
-a^2+b^2)^(1/4)/e^(1/2))/b^(11/2)/(-a^2+b^2)^(3/4)/d-9/8*(7*a^4-9*a^2*b^2+2
*b^4)*e^(11/2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2
))/b^(11/2)/(-a^2+b^2)^(3/4)/d+9/20*e^3*(7*a+2*b*cos(d*x+c))*(e*sin(d*x+c))
^(5/2)/b^3/d/(a+b*cos(d*x+c))+1/2*e*(e*sin(d*x+c))^(9/2)/b/d/(a+b*cos(d*x+c
))^2-3/4*a*(21*a^2-13*b^2)*e^6*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(
1/2)/b^6/d/(e*sin(d*x+c))^(1/2)+9/8*a*(7*a^4-9*a^2*b^2+2*b^4)*e^6*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1
/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^6/d/(a^
2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+9/8*a*(7*a^4-9*a^2*b^2+2*b^4
)*e^6*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/b^6/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-3/4*e^5*(21*a
^2-6*b^2-7*a*b*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^5/d
```

Rubi [A]
time = 1.02, antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2772, 2942, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{3ae^6(21a^2-13b^2)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{4b^6d\sqrt{e\sin(c+dx)}}-\frac{3e^5\sqrt{e\sin(c+dx)}\,(3(7a^2-2b^2)-7ab\cos(c+dx))}{4b^5d}-\frac{9e^{11/2}(7a^4-9a^2b^2+2b^4)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{11/2}d\,(b^2-a^2)^{3/4}}-\frac{9e^{11/2}(7a^4-9a^2b^2+2b^4)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{11/2}d\,(b^2-a^2)^{3/4}}-\frac{9ae^6(7a^4-9a^2b^2+2b^4)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8b^6d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}-\frac{9ae^6(7a^4-9a^2b^2+2b^4)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8b^6d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}+\frac{9e^3(e\sin(c+dx))^{5/2}(7a+2b\cos(c+dx))}{20b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-9*(7*a^4 - 9*a^2*b^2 + 2*b^4)*e^(11/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x
]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(11/2)*(-a^2 + b^2)^(3/4)*d) - (9*(
7*a^4 - 9*a^2*b^2 + 2*b^4)*e^(11/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/
((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*b^(11/2)*(-a^2 + b^2)^(3/4)*d) + (3*a*(21
*a^2 - 13*b^2)*e^6*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(4*
b^6*d*Sqrt[e*Sin[c + d*x]]) - (9*a*(7*a^4 - 9*a^2*b^2 + 2*b^4)*e^6*Elliptic
Pi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])
/(8*b^6*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (9*a*(7*
a^4 - 9*a^2*b^2 + 2*b^4)*e^6*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c -
```

```
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*b^6*(a^2 - b*(b + Sqrt[-a^2 + b^2]
))*d*Sqrt[e*Sin[c + d*x]]) - (3*e^5*(3*(7*a^2 - 2*b^2) - 7*a*b*Cos[c + d*x]
)*Sqrt[e*Sin[c + d*x]])/(4*b^5*d) + (9*e^3*(7*a + 2*b*Cos[c + d*x])*(e*Sin[
c + d*x])^(5/2))/(20*b^3*d*(a + b*Cos[c + d*x])) + (e*(e*Sin[c + d*x])^(9/2
))/(2*b*d*(a + b*Cos[c + d*x])^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
```

```
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
```

```
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{11/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(9e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^2}\,dx}{4b}$$

$$= \frac{9e^3(7a+2b\cos(c+dx))(e\sin(c+dx))^{5/2}}{20b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{9/2}}{2bd(a+b\cos(c+dx))^2} + \frac{(9e^4)\int}{}$$

$$= -\frac{3e^5(3(7a^2-2b^2)-7ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^5d} + \frac{9e^3(7a+2b\cos(c+}{20b^3d(a+b}$$

$$= -\frac{3e^5(3(7a^2-2b^2)-7ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^5d} + \frac{9e^3(7a+2b\cos(c+}{20b^3d(a+b}$$

$$= -\frac{3e^5(3(7a^2-2b^2)-7ab\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^5d} + \frac{9e^3(7a+2b\cos(c+}{20b^3d(a+b}$$

$$= \frac{3a(21a^2-13b^2)\,e^6F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{4b^6d\sqrt{e\sin(c+dx)}} - \frac{3e^5(3(7a^2-2b^2)-}{}$$

$$= \frac{3a(21a^2-13b^2)\,e^6F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{4b^6d\sqrt{e\sin(c+dx)}} - \frac{9a(7a^4-9a^2b^2+2b}{8b^6\,(a}$$

$$= -\frac{9(7a^4-9a^2b^2+2b^4)\,e^{11/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{11/2}\,(-a^2+b^2)^{3/4}\,d} - \frac{9(7a^4-9a^2b^2+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 44.74, size = 2024, normalized size = 3.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(11/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (((2*a*Cos[c + d*x])/b^4 + (-a^2 + b^2)^2/(2*b^5*(a + b*Cos[c + d*x])^2) -
(17*a*(a^2 - b^2))/(4*b^5*(a + b*Cos[c + d*x])) - Cos[2*(c + d*x)]/(5*b^3))
*Csc[c + d*x]^5*(e*Sin[c + d*x])^(11/2))/d + (3*(e*Sin[c + d*x])^(11/2)*((2
*(25*a^3 - 37*a*b^2)*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-
2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*Ar
cTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt
[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[
c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*
b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]
^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 -
b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c
+ d*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 +
Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(30*a^
2*b - 16*b^3)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*
Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/
4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
 + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d
*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*
a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^
2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^
2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 +
b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*
x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^
2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin
[c + d*x]^2)))))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2]) + ((-40*a^
2*b + 14*b^3)*Cos[c + d*x]*Cos[2*(c + d*x)]*(a + b*Sqrt[1 - Sin[c + d*x]^2]
)*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x
]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^
2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)
])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2
 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c
 + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sq
```

```
rt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*
b*Sin[c + d*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Sin[c + d*x]])/b -
(4*a*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/
4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin
[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1
[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a
^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(
-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*
Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x]^2])))/(40*b^5*d*
Sin[c + d*x]^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3725 vs. $2(621) = 1242$.
time = 0.72, size = 3726, normalized size = 6.17

method	result	size
default	Expression too large to display	3726

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2/5*e^3/b^3*(e*sin(d*x+c))^(5/2)-12*e^5/b^5*a^2*(e*sin(d*x+c))^(1/2)+4*e^5
/b^3*(e*sin(d*x+c))^(1/2)-19/4*e^7/b^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e
*sin(d*x+c))^(5/2)*a^4+21/4*e^7/b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(
d*x+c))^(5/2)*a^2-1/2*e^7*b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c)
)^(5/2)-15/4*e^9/b^5/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(1/2)
*a^6+8*e^9/b^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(1/2)*a^4-1
9/4*e^9/b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(1/2)*a^2+1/2*e^
9*b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(1/2)+63/16*e^7/b^5*(e
^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-
b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)*a^4-81/16*e^7/b^3*(e^2*(a^2-b^2)/b^
2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)
*(e*sin(d*x+c))^(1/2)+1)*a^2+9/8*e^7/b*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b
^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/
2)+1)+63/16*e^7/b^5*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arc
tan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)*a^4-81/16*e^7
/b^3*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)*a^2+9/8*e^7/b*(e^2*(a^2-b^2)
/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1
/4)*(e*sin(d*x+c))^(1/2)-1)+63/32*e^7/b^5*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^
2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c)
)^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2
)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))*a^4-81/32*
```

```
e^7/b^3*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c
)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2
)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/
2)+(e^2*(a^2-b^2)/b^2)^(1/2)))*a^2+9/16*e^7/b*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^
2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*
x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)*e^6*a*(1/b^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*
(10*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF
((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-7*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b
^2-2*b^2*cos(d*x+c)^2*sin(d*x+c))+3/b^6*(7*a^4-10*a^2*b^2+3*b^4)*(-1/2/b/(-
a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin
(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^2)^(1/
2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c
)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(
1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+3*(-5*a^6+11*a^4*b^2-7*a^2*b^4+
b^6)/b^6*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d
*x+c)^2*b^2+a^2)+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x
+c)+1)^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2
)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b
^2)^(1/2)/b),1/2*2^(1/2))+5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2
)^(1/2)/b),1/2*2^(1/2))-1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b
^2)^(1/2)/b),1/2*2^(1/2)))+4*a^2*(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/b^6*(1/4*b^2
/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)^
2+1/16*b^2*(13*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)/(-cos(d*x+c)^2*b^2+a^2)+13/32/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Ellip
ticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-45/64/(a^2-b^2)^2/b
/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-
sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+9/16/a^2/(a^2-b^2
)^2*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(11/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(11/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\sin\left(c+d\,x\right))^{11/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(11/2)/(a + b*cos(c + d*x))^3, x)
```

3.80 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=498

$$-\frac{7(5a^2-2b^2)\,e^{9/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}\,d}+\frac{7(5a^2-2b^2)\,e^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}\,d}+\frac{7a(5a}{}$$

```
[Out] -7/8*(5*a^2-2*b^2)*e^(9/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/b^(9/2)/(-a^2+b^2)^(1/4)/d+7/8*(5*a^2-2*b^2)*e^(9/2)*arctanh(
b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(9/2)/(-a^2+b^2)^(
1/4)/d+7/12*e^3*(5*a+2*b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/b^3/d/(a+b*cos(d*
x+c))+1/2*e*(e*sin(d*x+c))^(7/2)/b/d/(a+b*cos(d*x+c))^2-7/8*a*(5*a^2-2*b^2)
*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
Pi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(
1/2)/b^5/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-7/8*a*(5*a^2-2*b^2)*e^
5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(
cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2
)/b^5/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+35/4*a*e^4*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+
1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^4/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2942, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{7e^{9/2}(5a^2-2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}}+\frac{7e^{9/2}(5a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{9/2}d\sqrt[4]{b^2-a^2}}+\frac{7ae^5(5a^2-2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8b^5d\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{7ae^5(5a^2-2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8b^5d\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}-\frac{35ae^4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^4d\sqrt{\sin(c+dx)}}+\frac{7e^3(e\sin(c+dx))^{3/2}(5a+2b\cos(c+dx))}{12b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-7*(5*a^2 - 2*b^2)*e^(9/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*b^(9/2)*(-a^2 + b^2)^(1/4)*d) + (7*(5*a^2 - 2*b^2)
*e^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*b^(9/2)*(-a^2 + b^2)^(1/4)*d) + (7*a*(5*a^2 - 2*b^2)*e^5*EllipticPi[
(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8
*b^5*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (7*a*(5*a^2 - 2*b^2)*
e^5*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Si
n[c + d*x]])/(8*b^5*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (35*a*
e^4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(4*b^4*d*Sqrt[Si
n[c + d*x]]) + (7*e^3*(5*a + 2*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(12*
b^3*d*(a + b*Cos[c + d*x])) + (e*(e*Sin[c + d*x])^(7/2))/(2*b*d*(a + b*Cos[
c + d*x])^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(7e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^2}\,dx}{4b}$$

$$= \frac{7e^3(5a+2b\cos(c+dx))(e\sin(c+dx))^{3/2}}{12b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} + \frac{(7e^4)\,\int}{}$$

$$= \frac{7e^3(5a+2b\cos(c+dx))(e\sin(c+dx))^{3/2}}{12b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(35ae^4}{}$$

$$= \frac{7e^3(5a+2b\cos(c+dx))(e\sin(c+dx))^{3/2}}{12b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{7/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(7a(5a}{}$$

$$= -\frac{35ae^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^4d\sqrt{\sin(c+dx)}} + \frac{7e^3(5a+2b\cos(c+dx))(e\sin}{12b^3d(a+b\cos(c+}$$

$$= \frac{7a(5a^2-2b^2)\,e^5\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{8b^5\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{7a(5a}{}$$

$$= -\frac{7(5a^2-2b^2)\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{9/2}\sqrt[4]{-a^2+b^2}\,d} + \frac{7(5a^2-2b^2)\,e^{9/2}\tanh^{-1}\left(\right.}{8b^{9/2}\sqrt[4]{-a^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 34.49, size = 837, normalized size = 1.68

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^4*(e*Sin[c + d*x])^(9/2)*((2*Sin[c + d*x])/(3*b^3) + (11*a*Si
n[c + d*x])/(4*b^3*(a + b*Cos[c + d*x])) + (-(a^2*Sin[c + d*x]) + b^2*Sin[c
 + d*x])/(2*b^3*(a + b*Cos[c + d*x])^2)))/d - (7*(e*Sin[c + d*x])^(9/2)*((5
*a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqr
t[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]
*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqr
t[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2
```

```
- b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d
*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c +
d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))
/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*b
*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x
]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])
/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^
(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 +
I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqr
t[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b
^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a +
b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2]
)))/(8*b^3*d*Sin[c + d*x]^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3197 vs. $2(522) = 1044$.
time = 0.61, size = 3198, normalized size = 6.42

method	result	size
default	Expression too large to display	3198

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2/3*e^3/b^3*(e*sin(d*x+c))^(3/2)+13/4*e^5/b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2
)^2*(e*sin(d*x+c))^(7/2)*a^2-1/2*e^5*b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e
*sin(d*x+c))^(7/2)+9/4*e^7/b^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x
+c))^(3/2)*a^4-11/4*e^7/b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^
(3/2)*a^2+1/2*e^7*b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(3/2)-
35/32*e^5/b^5/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*a^2*ln((e*sin(d*x+c)-(e^2*(
a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))
/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*
(a^2-b^2)/b^2)^(1/2)))-35/16*e^5/b^5/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*a^2*
arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)-35/16*e^5/
b^5/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2
)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+7/16*e^5/b^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1
/2)*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)
+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(
d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+7/8*e^5/b^3/(e^2*(a^2-b^2
)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c)
)^(1/2)+1)+7/8*e^5/b^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^
2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)-(cos(d*x+c)^2*e*sin(d*x+c))^
(1/2)*e^5*a*(-3/b^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(2*EllipticE((-sin(d*x+c)+1)^(1/2)
,1/2*2^(1/2))-EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)))-2*(5*a^2-3*b^2)
```

```
/b^4*(-1/2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin
(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(-sin(d*x+c)
+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2
+b^2)^(1/2)/b),1/2*2^(1/2)))+(11*a^4-14*a^2*b^2+3*b^4)/b^4*(1/2*b^2/e/a^2/(
a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^
2)-1/2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1
/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1
)^(1/2),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(
1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/
2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticP
i((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/8/(a^2-b^2)
/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)
^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)
+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2
+b^2)^(1/2)/b),1/2*2^(1/2)))-4*a^2*(a^4-2*a^2*b^2+b^4)/b^4*(1/4*b^2/e/a^2/(
a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^
2)^2+1/16*b^2*(11*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*sin(d*x+c)*(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-11/16/a^2/(a^2-b^2)^2*(-sin(d*x+c)
+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/8/a^4/(a^2-b^2)^2*(
-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*
e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2+11/32/
a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^
(1/2))-3/16/a^4/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*si
n(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))*b^2-21/64/(a^2-b^2)^2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+
b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2
*2^(1/2))+7/16/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*
EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16
/a^4/(a^2-b^2)^2*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi
((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-21/64/(a^2-b^2
)^2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)
```

```
+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(9/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\sin(c+d\,x))^{9/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(9/2)/(a + b*cos(c + d*x))^3, x)
```

3.81 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=512

$$\frac{5(3a^2-2b^2)\,e^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d}+\frac{5(3a^2-2b^2)\,e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d}-\frac{15ae^4}{}$$

```
[Out] 5/8*(3*a^2-2*b^2)*e^(7/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)^(3/4)/d+5/8*(3*a^2-2*b^2)*e^(7/2)*arctanh(b
^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(7/2)/(-a^2+b^2)^(3
/4)/d+1/2*e*(e*sin(d*x+c))^(5/2)/b/d/(a+b*cos(d*x+c))^2+15/4*a*e^4*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1
/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/b^4/d/(e*sin(d*x+c))^(1/2)-5/8*a*(
3*a^2-2*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*
sin(d*x+c)^(1/2)/b^4/d/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-5/
8*a*(3*a^2-2*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+
1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1
/2))*sin(d*x+c)^(1/2)/b^4/d/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/
2)+5/4*e^3*(3*a+2*b*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/b^3/d/(a+b*cos(d*x+c))
```

Rubi [A]
time = 0.71, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2942, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{5e^{7/2}(3a^2-2b^2)\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}}+\frac{5e^{7/2}(3a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{7/2}d\left(b^2-a^2\right)^{3/4}}+\frac{5ae^4(3a^2-2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)}{8b^4d\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}+\frac{5ae^4(3a^2-2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)}{8b^4d\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}-\frac{15ae^4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)}{4b^4d\sqrt{e\sin(c+dx)}}+\frac{5e^3\sqrt{e\sin(c+dx)}\,(3a+2b\cos(c+dx))}{4b^3d(a+b\cos(c+dx))}+\frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (5*(3*a^2 - 2*b^2)*e^(7/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*b^(7/2)*(-a^2 + b^2)^(3/4)*d) + (5*(3*a^2 - 2*b^2)*
e^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])
])/(8*b^(7/2)*(-a^2 + b^2)^(3/4)*d) - (15*a*e^4*EllipticF[(c - Pi/2 + d*x)/
2, 2]*Sqrt[Sin[c + d*x]])/(4*b^4*d*Sqrt[e*Sin[c + d*x]]) + (5*a*(3*a^2 - 2*
b^2)*e^4*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(8*b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c +
d*x]]) + (5*a*(3*a^2 - 2*b^2)*e^4*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*b^4*(a^2 - b*(b + Sqrt[-a^2 +
 b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (5*e^3*(3*a + 2*b*Cos[c + d*x])*Sqrt[e*Si
n[c + d*x]])/(4*b^3*d*(a + b*Cos[c + d*x])) + (e*(e*Sin[c + d*x])^(5/2))/(2
*b*d*(a + b*Cos[c + d*x])^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(5e^2)\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^2}\,dx}{4b}$$

$$= \frac{5e^3(3a+2b\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} + \frac{(5e^4)\int -}{}$$

$$= \frac{5e^3(3a+2b\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(15ae^4)\int}{}$$

$$= \frac{5e^3(3a+2b\cos(c+dx))\sqrt{e\sin(c+dx)}}{4b^3d(a+b\cos(c+dx))} + \frac{e(e\sin(c+dx))^{5/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(5a(3a^2 -}{}$$

$$= -\frac{15ae^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{4b^4d\sqrt{e\sin(c+dx)}} + \frac{5e^3(3a+2b\cos(c+dx))\sqrt{e\sin(}}{4b^3d(a+b\cos(c+dx))}$$

$$= -\frac{15ae^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{4b^4d\sqrt{e\sin(c+dx)}} + \frac{5a(3a^2-2b^2)\,e^4\Pi\left(\frac{2b}{b-\sqrt{-a^2+b}}\right.}{8b^4\left(a^2-b\left(b-\sqrt{-a}\right.\right.}$$

$$= \frac{5(3a^2-2b^2)\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{7/2}\left(-a^2+b^2\right)^{3/4}d} + \frac{5(3a^2-2b^2)\,e^{7/2}\tanh^{-1}\left(\sqrt{}\right.}{8b^{7/2}\left(-a^2+b^2\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 38.83, size = 946, normalized size = 1.85

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(e*Sin[c + d*x])^(7/2)*(7*a^2 + 2*b^2 + 9*a*b*Cos[c + d*x]
+ ((a + b*Cos[c + d*x])*(-6*b - 7*a*Cos[c + d*x] + 4*b*Cos[2*(c + d*x)])*(8
*(a + b) - 5*(3*a + 2*b)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[(c + d*x)/2]^2, ((
-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sqrt[Sec[(c + d*x)/2]^2] + (3*a - 2*b)
*AppellF1[5/4, 1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]
^2)/(a + b)]*Sqrt[Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2))/((Sqrt[Sec[(c +
d*x)/2]^2]*Sin[c + d*x]*Tan[(c + d*x)/2]*(-45*(3*a + 2*b)*AppellF1[1/4, 1/2
, 1, 5/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)] + 18*
```

```
(3*a - 2*b)*AppellF1[5/4, 1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(
c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2 + (9*(3*a + 2*b)*(2*(a - b)*Appe
llF1[5/4, 1/2, 2, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(
a + b)] + (a + b)*AppellF1[5/4, 3/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)
*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2)/(a + b) + 9*(3*a - 2*b)*
AppellF1[5/4, 1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^
2)/(a + b)]*Tan[(c + d*x)/2]^2 - (5*(3*a - 2*b)*(2*(a - b)*AppellF1[9/4, 1/
2, 2, 13/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)] + (
a + b)*AppellF1[9/4, 3/2, 1, 13/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Tan[(c +
d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)/(a + b)))/9 + C
os[c + d*x]*(8*(a + b) - 5*(3*a + 2*b)*AppellF1[1/4, 1/2, 1, 5/4, -Tan[(c +
 d*x)/2]^2, ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sqrt[Sec[(c + d*x)/2]^2]
 + (3*a - 2*b)*AppellF1[5/4, 1/2, 1, 9/4, -Tan[(c + d*x)/2]^2, ((-a + b)*Ta
n[(c + d*x)/2]^2)/(a + b)]*Sqrt[Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2))))/
(4*b^3*d*(a + b*Cos[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3145 vs. $2(536) = 1072$.
time = 0.64, size = 3146, normalized size = 6.14

method	result	size
default	Expression too large to display	3146

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2*e^3/b^3*(e*sin(d*x+c))^(1/2)+11/4*e^5/b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^
2*(e*sin(d*x+c))^(5/2)*a^2-1/2*e^5*b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*s
in(d*x+c))^(5/2)+7/4*e^7/b^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c
))^(1/2)*a^4-9/4*e^7/b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(1/
2)*a^2+1/2*e^7*b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(1/2)-15/
32*e^5/b^3*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*
x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/
b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^
(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))*a^2+5/16*e^5/b*(e^2*(a^2-b^2)/b^2)^(1/4)/
(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin
(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b
^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-15/
16*e^5/b^3*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/
2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)*a^2+5/8*e^5/b*(e^2*(a^
2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b
^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)-15/16*e^5/b^3*(e^2*(a^2-b^2)/b^2)^(1/4)/(
a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*
x+c))^(1/2)-1)*a^2+5/8*e^5/b*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^
(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)-(cos
```

```
(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^4*a*(-3/b^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*(-5*a^2+3*b^2)/b^4*(-1/2/b/(-a^2+b^2)^
(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*
x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1
)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^2)^(1/2)*(-sin(
d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin
(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1
+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+1/b^4*(11*a^4-14*a^2*b^2+3*b^4)*(1/2*b^2
/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)+
1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*
2^(1/2))-5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2
)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^
(1/2))+1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b
^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*
2^(1/2))+5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2
)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^
(1/2))-1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b
^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*
2^(1/2)))-4*a^2*(a^4-2*a^2*b^2+b^4)/b^4*(1/4*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c
)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)^2+1/16*b^2*(13*a^2-6*b^2)/a
^4/(a^2-b^2)^2/e*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)+
13/32/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))*b^2-45/64/(a^2-b^2)^2/b/(-a^2+b^2)^(1/2)*(-sin(d*x
+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*
x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-
a^2+b^2)^(1/2)/b),1/2*2^(1/2))+9/16/a^2/(a^2-b^2)^2*b/(-a^2+b^2)^(1/2)*(-si
n(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/
(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^3/(-a^2+b^2)^(1/
2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c
)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(
1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+45/64/(a^2-b^2)^2/b/(-a^2+b^2)^(
1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)
^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-9/16/a^2/(a^2-b^2)^2*b/(-a^2+b
^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(co
```

```
s(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+
c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+3/16/a^4/(a^2-b^2)^2*b^3/
(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] e^(7/2)*integrate(sin(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*cos(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\sin(c+d\,x))^{7/2}}{(a+b\,\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(7/2)/(a + b*cos(c + d*x))^3, x)
```

3.82 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=520

$$-\frac{3(a^2-2b^2)\,e^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d}+\frac{3(a^2-2b^2)\,e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{5/2}\left(-a^2+b^2\right)^{5/4}d}-\frac{3a(a^2}{}$$

```
[Out] -3/8*(a^2-2*b^2)*e^(5/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/
4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(5/4)/d+3/8*(a^2-2*b^2)*e^(5/2)*arctanh(b^(1
/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(5/2)/(-a^2+b^2)^(5/4)
/d+1/2*e*(e*sin(d*x+c))^(3/2)/b/d/(a+b*cos(d*x+c))^2-3/4*a*e*(e*sin(d*x+c))
^(3/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))+3/8*a*(a^2-2*b^2)*e^3*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+
1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^3/(a^2-b^2)/d
/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+3/8*a*(a^2-2*b^2)*e^3*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/
4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^3/(a^2-b
^2)/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-3/4*a*e^2*(sin(1/2*c+1/4*Pi
+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2
*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.72, antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2943, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{3e^{5/2}(a^2-2b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{5/2}d\,(b^2-a^2)^{5/4}}+\frac{3ae^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^2d\,(a^2-b^2)\sqrt{\sin(c+dx)}}-\frac{3ae(e\sin(c+dx))^{3/2}}{4bd\,(a^2-b^2)\,(a+b\cos(c+dx))}+\frac{3e^{5/2}(a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{5/2}d\,(b^2-a^2)^{5/4}}-\frac{3ae^3(a^2-2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8b^3d\,(a^2-b^2)\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{3ae^3(a^2-2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8b^3d\,(a^2-b^2)\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}+\frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-3*(a^2 - 2*b^2)*e^(5/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^
2)^(1/4)*Sqrt[e])])/(8*b^(5/2)*(-a^2 + b^2)^(5/4)*d) + (3*(a^2 - 2*b^2)*e^(
5/2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/
(8*b^(5/2)*(-a^2 + b^2)^(5/4)*d) - (3*a*(a^2 - 2*b^2)*e^3*EllipticPi[(2*b)/
(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*b^3*(
a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) - (3*a*(a^2 - 2*b
^2)*e^3*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqr
t[Sin[c + d*x]])/(8*b^3*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c +
 d*x]]) + (3*a*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(
4*b^2*(a^2 - b^2)*d*Sqrt[Sin[c + d*x]]) + (e*(e*Sin[c + d*x])^(3/2))/(2*b*d
*(a + b*Cos[c + d*x])^2) - (3*a*e*(e*Sin[c + d*x])^(3/2))/(4*b*(a^2 - b^2)*
d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
```

```
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} - \frac{(3e^2)\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}\,dx}{4b}$$

$$= \frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} - \frac{3ae(e\sin(c+dx))^{3/2}}{4b(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{(3e^2)\int \frac{(b+\frac{1}{2}a\cos(c+}{}}{4b}$$

$$= \frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} - \frac{3ae(e\sin(c+dx))^{3/2}}{4b(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{(3ae^2)\int \sqrt{e\sin(c}}{8b^2(a^2-}$$

$$= \frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} - \frac{3ae(e\sin(c+dx))^{3/2}}{4b(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{(3a(a^2-2b^2)\,e^3)}{}$$

$$= \frac{3ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4b^2(a^2-b^2)\,d\sqrt{\sin(c+dx)}} + \frac{e(e\sin(c+dx))^{3/2}}{2bd(a+b\cos(c+dx))^2} - \frac{3}{4b(a^2-}$$

$$= -\frac{3a(a^2-2b^2)\,e^3\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{8b^3(a^2-b^2)\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{3a(a^2}{}$$

$$= -\frac{3(a^2-2b^2)\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8b^{5/2}(-a^2+b^2)^{5/4}\,d} + \frac{3(a^2-2b^2)\,e^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}}{}\right.}{8b^{5/2}(-a^2+b^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 32.84, size = 831, normalized size = 1.60

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*(Sin[c + d*x]/(2*b*(a + b*Cos[c + d*
x])^2) + (3*a*Sin[c + d*x])/(4*b*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d + (
3*(e*Sin[c + d*x])^(5/2)*((a*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*
(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*A
rcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqr
t[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin
[c + d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[S
```

```
in[c + d*x]] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin
[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*
Sqrt[1 - Sin[c + d*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1
 - Sin[c + d*x]^2)) + (4*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I
)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*S
qrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1
+ I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Lo
g[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2,
1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/
2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[c + d*x]
)*Sqrt[1 - Sin[c + d*x]^2])))/(8*(a - b)*b*(a + b)*d*Sin[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3100 vs. $2(544) = 1088$.
time = 0.60, size = 3101, normalized size = 5.96

method	result	size
default	Expression too large to display	3101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-5/4*e^3*b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2/(a^2-b^2)*(e*sin(d*x+c))^(7/2
)*a^2+1/2*e^3*b^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2/(a^2-b^2)*(e*sin(d*x+c)
)^(7/2)-1/4*e^5/b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(3/2)*a^
2+1/2*e^5*b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(3/2)+3/16*e^3
/b^3/(a^2-b^2)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b
^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)*a^2-3/8*e^3/b/(a^2-b^2)/(e^2*(a^2-b^
2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c
))^(1/2)-1)+3/32*e^3/b^3/(a^2-b^2)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln((e*
sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2
-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1
/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))*a^2-3/16*e^3/b/(a^2-b^2)/(e^2*(a^2-
b^2)/b^2)^(1/4)*2^(1/2)*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d
*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2
)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+3/16*
e^3/b^3/(a^2-b^2)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^
2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)*a^2-3/8*e^3/b/(a^2-b^2)/(e^2*(a^2
-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*
x+c))^(1/2)+1)-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^3*a*(3/b^2*(-1/2/b^2*(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1
/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+
```

b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))-(7*a^2-3*b^2)/b^2*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-1/2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+4*a^2*(a^2-b^2)/b^2*(1/4*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)^2+1/16*b^2*(11*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-11/16/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/8/a^4/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2+11/32/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-21/64/(a^2-b^2)^2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+7/16/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-21/64/(a^2-b^2)^2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+7/16/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin...

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] e^(5/2)*integrate(sin(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\sin\left(c+d\,x\right))^{5/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3, x)
```

3.83 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=534

$$-\frac{(a^2+2b^2)\,e^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{3/2}\left(-a^2+b^2\right)^{7/4}d}-\frac{(a^2+2b^2)\,e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{3/2}\left(-a^2+b^2\right)^{7/4}d}-\frac{ae^2F\left(\frac{1}{2}(c\right.}{4b^2\,(a}$$

```
[Out] -1/8*(a^2+2*b^2)*e^(3/2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/
4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(7/4)/d-1/8*(a^2+2*b^2)*e^(3/2)*arctanh(b^(1
/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/b^(3/2)/(-a^2+b^2)^(7/4)
/d+1/4*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/b^2/(a^2-b^2)
/d/(e*sin(d*x+c))^(1/2)-1/8*a*(a^2+2*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(
b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/(a^2-b^2)/d/(a^2-b*(b-(-a
^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-1/8*a*(a^2+2*b^2)*e^2*(sin(1/2*c+1/4*P
i+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1
/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/b^2/(a^2-b^2)/d/
(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+1/2*e*(e*sin(d*x+c))^(1/2
)/b/d/(a+b*cos(d*x+c))^2-1/4*a*e*(e*sin(d*x+c))^(1/2)/b/(a^2-b^2)/d/(a+b*co
s(d*x+c))
```

Rubi [A]
time = 0.76, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2772, 2943, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{e^{3/2}(a^2+2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{3/2}d\,(b^2-a^2)^{7/4}}-\frac{ae^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{4b^2d\,(a^2-b^2)\,\sqrt{e\sin(c+dx)}}+\frac{ae^2(a^2+2b^2)\,\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8b^2d\,(a^2-b^2)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}+\frac{ae^2(a^2+2b^2)\,\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8b^2d\,(a^2-b^2)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}-\frac{ae\sqrt{e\sin(c+dx)}}{4bd\,(a^2-b^2)\,(a+b\cos(c+dx))}-\frac{e^{3/2}(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8b^{3/2}d\,(b^2-a^2)^{7/4}}+\frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -1/8*((a^2 + 2*b^2)*e^(3/2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(b^(3/2)*(-a^2 + b^2)^(7/4)*d) - ((a^2 + 2*b^2)*e^(3/
2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8
*b^(3/2)*(-a^2 + b^2)^(7/4)*d) - (a*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(4*b^2*(a^2 - b^2)*d*Sqrt[e*Sin[c + d*x]]) + (a*(a^2 + 2*
b^2)*e^2*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(8*b^2*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sqr
t[e*Sin[c + d*x]]) + (a*(a^2 + 2*b^2)*e^2*EllipticPi[(2*b)/(b + Sqrt[-a^2 +
 b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*b^2*(a^2 - b^2)*(a^2
 - b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) + (e*Sqrt[e*Sin[c + d*x
```

```
]])/(2*b*d*(a + b*Cos[c + d*x])^2) - (a*e*Sqrt[e*Sin[c + d*x]])/(4*b*(a^2 -
 b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\cos(c+dx))^3}\,dx = \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{e^2\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx}{4b}$$

$$= \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{ae\sqrt{e\sin(c+dx)}}{4b\,(a^2-b^2)\,d(a+b\cos(c+dx))} - \frac{e^2\int \frac{-b+\cdots}{(a+b\cos(c+dx)\cdots}}{4b\,(\cdots}$$

$$= \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{ae\sqrt{e\sin(c+dx)}}{4b\,(a^2-b^2)\,d(a+b\cos(c+dx))} - \frac{(ae^2)\int \frac{1}{\sqrt{e\sin(\cdots}}}{8b^2\,(a^2-\cdots}$$

$$= \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{ae\sqrt{e\sin(c+dx)}}{4b\,(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{(a(a^2+2b^2)\,e^2)\cdots}{\cdots}$$

$$= -\frac{ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{4b^2\,(a^2-b^2)\,d\sqrt{e\sin(c+dx)}} + \frac{e\sqrt{e\sin(c+dx)}}{2bd(a+b\cos(c+dx))^2} - \frac{\cdots}{4b\,(a^2\cdots}$$

$$= -\frac{ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{4b^2\,(a^2-b^2)\,d\sqrt{e\sin(c+dx)}} - \frac{a(a^2+2b^2)\,e^2\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\cdots\right.}{8b^2\,(-a^2+b^2)^{3/2}\left(b-\sqrt{-\cdots}\right.}$$

$$= -\frac{(a^2+2b^2)\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8b^{3/2}\,(-a^2+b^2)^{7/4}\,d} - \frac{(a^2+2b^2)\,e^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt[4]{\cdots}}\right.}{8b^{3/2}\,(-a^2+b^2\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 29.16, size = 1211, normalized size = 2.27

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((1/(2*b*(a + b*Cos[c + d*x])^2) + a/(4*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])
))*Csc[c + d*x]*(e*Sin[c + d*x])^(3/2))/d - ((e*Sin[c + d*x])^(3/2)*((2*a*C
os[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*
Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt
[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*
Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a
^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c
```

```
+ d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1
[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt
[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/
2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*Ap
pellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)
] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x
]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((
a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) - (4*b*Cos[c + d*x]*(a + b*Sqrt[1
 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*S
qrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt
[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b
]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] - Log[Sqrt[-a^2
 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c
 + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4,
 Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*x]])/(Sq
rt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*
x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9
/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*Appel
lF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*S
in[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c + d*x])*S
qrt[1 - Sin[c + d*x]^2])))/(8*(a - b)*b*(a + b)*d*Sin[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3064 vs. $2(558) = 1116$.
time = 0.61, size = 3065, normalized size = 5.74

method	result	size
default	Expression too large to display	3065

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/4*e^3*b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2/(a^2-b^2)*(e*sin(d*x+c))^(5/2
)*a^2+1/2*e^3*b^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2/(a^2-b^2)*(e*sin(d*x+c)
)^(5/2)+1/4*e^5/b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(1/2)*a^
2+1/2*e^5*b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(1/2)-1/16*e^3
/b/(a^2-b^2)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(
1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)*a^2-1/8*e^3*b/(a^2-b
^2)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2
*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)-1/16*e^3/b/(a^2-b^2)*(e^2*(a^
2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b
^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)*a^2-1/8*e^3*b/(a^2-b^2)*(e^2*(a^2-b^2)/b^
2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)
*(e*sin(d*x+c))^(1/2)-1)-1/32*e^3/b/(a^2-b^2)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^
2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*
```

x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))*a^2-1/
16*e^3*b/(a^2-b^2)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((
e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a
^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^
(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)
*e^2*a*(3/b^2*(-1/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(
1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/
2))+1/2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*Elli
pticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+(-7*a^2
+3*b^2)/b^2*(1/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-co
s(d*x+c)^2*b^2+a^2)+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(
d*x+c)+1)^(1/2),1/2*2^(1/2))-5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+
b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^
2+b^2)^(1/2)/b),1/2*2^(1/2))+5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+
b^2)^(1/2)/b),1/2*2^(1/2))-1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^
2+b^2)^(1/2)/b),1/2*2^(1/2)))+4*a^2*(a^2-b^2)/b^2*(1/4*b^2/e/a^2/(a^2-b^2)*
(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)^2+1/16*b^2*(13*a^
2-6*b^2)/a^4/(a^2-b^2)^2/e*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2
b^2+a^2)+13/32/a^2/(a^2-b^2)^2(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)
+1)^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-45/64/(a^2-b^2)^2/b/(-a^2+b^2)^(1/2)
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/
2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+9/16/a^2/(a^2-b^2)^2*b/(-a^2+b^2)^
(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*
x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1
)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^3/(-a^
2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/
(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d
*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+45/64/(a^2-b^2)^2/b/(-
a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2

```
)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin
(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-9/16/a^2/(a^2-b^2)^2
*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi(
(-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+3/16/a^4/(a^2-b
^2)^2*b^3/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*Elli
pticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*
x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] e^(3/2)*integrate(sin(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,\sin\left(c+d\,x\right))^{3/2}}{\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((e*sin(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3, x)
```

3.84 $\int \frac{\sqrt{e \sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=529

$$-\frac{(3a^2+2b^2)\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\,(-a^2+b^2)^{9/4}\,d}+\frac{(3a^2+2b^2)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\,(-a^2+b^2)^{9/4}\,d}+\frac{a(3a^2+}{}$$

```
[Out] -1/2*b*(e*sin(d*x+c))^(3/2)/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2-5/4*a*b*(e*sin
(d*x+c))^(3/2)/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))-1/8*(3*a^2+2*b^2)*arctan(b^
(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(9/
4)/d/b^(1/2)+1/8*(3*a^2+2*b^2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b
^2)^(1/4)/e^(1/2))*e^(1/2)/(-a^2+b^2)^(9/4)/d/b^(1/2)-1/8*a*(3*a^2+2*b^2)*e
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(c
os(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)
/b/(a^2-b^2)^2/d/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-1/8*a*(3*a^2+2*b
^2)*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/b/(a^2-b^2)^2/d/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-5/4*a*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*
c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/sin(d*x+c)^(1
/2)
```

Rubi [A]
time = 0.76, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2943, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{\sqrt{e}\,(3a^2+2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8\sqrt{b}\,d\,(b^2-a^2)^{9/4}}+\frac{\sqrt{e}\,(3a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{b^2-a^2}}\right)}{8\sqrt{b}\,d\,(b^2-a^2)^{9/4}}-\frac{5ab(e\sin(c+dx))^{3/2}}{4de\,(a^2-b^2)^2\,(a+b\cos(c+dx))}-\frac{b(e\sin(c+dx))^{3/2}}{2de\,(a^2-b^2)\,(a+b\cos(c+dx))^2}+\frac{5aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4d\,(a^2-b^2)^2\sqrt{\sin(c+dx)}}+\frac{ae(3a^2+2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8bd\,(a^2-b^2)^2\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{ae(3a^2+2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8bd\,(a^2-b^2)^2\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -1/8*((3*a^2 + 2*b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[e])])/(Sqrt[b]*(-a^2 + b^2)^(9/4)*d) + ((3*a^2 + 2*b^2)*S
qrt[e]*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])]
)/(8*Sqrt[b]*(-a^2 + b^2)^(9/4)*d) + (a*(3*a^2 + 2*b^2)*e*EllipticPi[(2*b)/
(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*b*(a^
2 - b^2)^2*(b - Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d*x]]) + (a*(3*a^2 + 2*b
^2)*e*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/(8*b*(a^2 - b^2)^2*(b + Sqrt[-a^2 + b^2])*d*Sqrt[e*Sin[c + d
*x]]) + (5*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(4*(a^2
 - b^2)^2*d*Sqrt[Sin[c + d*x]]) - (b*(e*Sin[c + d*x])^(3/2))/(2*(a^2 - b^2)
```

```
*d*e*(a + b*Cos[c + d*x])^2) - (5*a*b*(e*Sin[c + d*x])^(3/2))/(4*(a^2 - b^2
)^2*d*e*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = -\frac{b(e\sin(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2} - \frac{\int \frac{\left(-2a+\frac{1}{2}b\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{(a+b\cos(c+dx))^2}dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b(e\sin(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2} - \frac{5ab(e\sin(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))} + \frac{\int (\ldots}{\ldots}$$

$$= -\frac{b(e\sin(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2} - \frac{5ab(e\sin(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))} + \frac{(5a\ldots}{\ldots}$$

$$= -\frac{b(e\sin(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2} - \frac{5ab(e\sin(c+dx))^{3/2}}{4\left(a^2-b^2\right)^2 de(a+b\cos(c+dx))} - \frac{(a(\ldots}{\ldots}$$

$$= \frac{5aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4\left(a^2-b^2\right)^2 d\sqrt{\sin(c+dx)}} - \frac{b(e\sin(c+dx))^{3/2}}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2} \ldots$$

$$= \frac{a\left(3a^2+2b^2\right)e\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{8b\left(a^2-b^2\right)^2\left(b-\sqrt{-a^2+b^2}\right)d\sqrt{e\sin(c+dx)}} + \frac{a\left(3a^2+\ldots\right.}{\ldots}$$

$$= -\frac{\left(3a^2+2b^2\right)\sqrt{e}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\sqrt{b}\left(-a^2+b^2\right)^{9/4}d} + \frac{\left(3a^2+2b^2\right)\sqrt{e}\tanh^{-1}\left(\ldots\right.}{8\sqrt{b}\left(-a^2+\ldots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 21.77, size = 748, normalized size = 1.41

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Sqrt[e*Sin[c + d*x]]*((-2*b*(7*a^2 - 2*b^2 + 5*a*b*Cos[c + d*x])*Sin[c + d
*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (Cos[c + d*x]*(a + b*Sqrt[Cos
[c + d*x]^2])*((5*a*Sec[c + d*x]*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1
 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (
Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2
```

```
] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]
+ Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]
] + b*Sin[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2
, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2)))/(Sqrt[b]*(-a^2 +
b^2)) + (48*(4*a^2 + b^2)*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt
[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Si
n[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(
-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 +
b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c +
d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c
+ d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 -
b^2))))/Sqrt[Cos[c + d*x]^2]))/(12*(a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])
*Sqrt[Sin[c + d*x]])))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2889 vs. $2(553) = 1106$.
time = 0.55, size = 2890, normalized size = 5.46

method	result	size
default	Expression too large to display	2890

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/4*e*b^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2/(a^4-2*a^2*b^2+b^4)*(e*sin(d*
x+c))^(7/2)*a^2-1/2*e*b^5/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2/(a^4-2*a^2*b^2+
b^4)*(e*sin(d*x+c))^(7/2)-7/4*e^3*b/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2/(a^2-
b^2)*(e*sin(d*x+c))^(3/2)*a^2-1/2*e^3*b^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2
/(a^2-b^2)*(e*sin(d*x+c))^(3/2)-3/32*e/b/(a^4-2*a^2*b^2+b^4)/(e^2*(a^2-b^2)
/b^2)^(1/4)*2^(1/2)*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c
))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^
2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))*a^2-1/16*
e*b/(a^4-2*a^2*b^2+b^4)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln((e*sin(d*x+c)-
(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^
(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)
+(e^2*(a^2-b^2)/b^2)^(1/2)))-3/16*e/b/(a^4-2*a^2*b^2+b^4)/(e^2*(a^2-b^2)/b^
2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1
/2)+1)*a^2-1/8*e*b/(a^4-2*a^2*b^2+b^4)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)-3/16*e/b/(a^
4-2*a^2*b^2+b^4)/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2
-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)*a^2-1/8*e*b/(a^4-2*a^2*b^2+b^4)/(e
^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e
*sin(d*x+c))^(1/2)-1)-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e*a*(3/2*b^2/e/a^2/
(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a
^2)-3/2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
```

```
c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))+3/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))-9/8/(a^2-b^2)/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^
(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1
/2))+3/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*Elliptic
Pi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-9/8/(a^2-b^2
)/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*
x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1
)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+3/4/a^2/(a^2-b^2)*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^
2+b^2)^(1/2)/b),1/2*2^(1/2))-4*a^2*(1/4*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)^2+1/16*b^2*(11*a^2-6*
b^2)/a^4/(a^2-b^2)^2/e*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d
*x+c)^2*b^2+a^2)-11/16/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin
(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/8/a^4/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Elli
pticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2+11/32/a^2/(a^2-b^2)^2*(-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^2
)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-2
1/64/(a^2-b^2)^2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi
((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+7/16/a^2/(a^2-
b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d
*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+
1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^2*(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1
/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-21/64/(a^2-b^2)^2/b^2*(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2
)^(1/2)/b),1/2*2^(1/2))+7/16/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+
b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2
*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)
/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))
)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
[Out] e^(1/2)*integrate(sqrt(sin(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*cos(d*x+c))**3,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
[Out] integrate(e^(1/2)*sqrt(sin(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,\sin\left(c+d\,x\right)}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3, x)
```

3.85 $\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=535

$$\frac{3\sqrt{b}\,(5a^2+2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\,(-a^2+b^2)^{11/4}\,d\sqrt{e}}+\frac{3\sqrt{b}\,(5a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\,(-a^2+b^2)^{11/4}\,d\sqrt{e}}-\frac{7aF(}{4}$$

```
[Out] 3/8*(5*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1
/2))*b^(1/2)/(-a^2+b^2)^(11/4)/d/e^(1/2)+3/8*(5*a^2+2*b^2)*arctanh(b^(1/2)*
(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))*b^(1/2)/(-a^2+b^2)^(11/4)/d/
e^(1/2)+7/4*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/
d/(e*sin(d*x+c))^(1/2)-3/8*a*(5*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(
-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a^2-b*(b-(-a^2+b^
2)^(1/2)))/(e*sin(d*x+c))^(1/2)-3/8*a*(5*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x)
,2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a^2-b*(b
+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)-1/2*b*(e*sin(d*x+c))^(1/2)/(a^2-b^
2)/d/e/(a+b*cos(d*x+c))^2-7/4*a*b*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/d/e/(a+b
*cos(d*x+c))
```

Rubi [A]
time = 0.80, antiderivative size = 535, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2773, 2943, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{3\sqrt{b}\,(5a^2+2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8d\sqrt{e}\,(b^2-a^2)^{11/4}}+\frac{3\sqrt{b}\,(5a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8d\sqrt{e}\,(b^2-a^2)^{11/4}}-\frac{7ab\sqrt{e\sin(c+dx)}}{4de\,(a^2-b^2)^2\,(a+b\cos(c+dx))}-\frac{b\sqrt{e\sin(c+dx)}}{2de\,(a^2-b^2)\,(a+b\cos(c+dx))^2}-\frac{7a\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{4d\,(a^2-b^2)^2\sqrt{e\sin(c+dx)}}+\frac{3a(5a^2+2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8d\,(a^2-b^2)^2\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}+\frac{3a(5a^2+2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8d\,(a^2-b^2)^2\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (3*Sqrt[b]*(5*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(11/4)*d*Sqrt[e]) + (3*Sqrt[b]*(5*a^2
+ 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*(-a^2 + b^2)^(11/4)*d*Sqrt[e]) - (7*a*EllipticF[(c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(4*(a^2 - b^2)^2*d*Sqrt[e*Sin[c + d*x]]) + (3*a*(5*a
^2 + 2*b^2)*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]
*Sqrt[Sin[c + d*x]])/(8*(a^2 - b^2)^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*Sq
rt[e*Sin[c + d*x]]) + (3*a*(5*a^2 + 2*b^2)*EllipticPi[(2*b)/(b + Sqrt[-a^2
+ b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*(a^2 - b^2)^2*(a^2 -
 b*(b + Sqrt[-a^2 + b^2]))*d*Sqrt[e*Sin[c + d*x]]) - (b*Sqrt[e*Sin[c + d*x]
```

```
])/(2*(a^2 - b^2)*d*e*(a + b*Cos[c + d*x])^2) - (7*a*b*Sqrt[e*Sin[c + d*x]]
)/(4*(a^2 - b^2)^2*d*e*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{e\sin(c+dx)}}\,dx = -\frac{b\sqrt{e\sin(c+dx)}}{2(a^2-b^2)\,de(a+b\cos(c+dx))^2} - \frac{\int \frac{-2a+\frac{3}{2}b\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{e\sin(c+\cdots}}}{2(a^2-b^2)}$$

$$= -\frac{b\sqrt{e\sin(c+dx)}}{2(a^2-b^2)\,de(a+b\cos(c+dx))^2} - \frac{7ab\sqrt{e\sin(c+dx)}}{4(a^2-b^2)^2\,de(a+b\cos(\cdots}$$

$$= -\frac{b\sqrt{e\sin(c+dx)}}{2(a^2-b^2)\,de(a+b\cos(c+dx))^2} - \frac{7ab\sqrt{e\sin(c+dx)}}{4(a^2-b^2)^2\,de(a+b\cos(\cdots}$$

$$= -\frac{b\sqrt{e\sin(c+dx)}}{2(a^2-b^2)\,de(a+b\cos(c+dx))^2} - \frac{7ab\sqrt{e\sin(c+dx)}}{4(a^2-b^2)^2\,de(a+b\cos(\cdots}$$

$$= -\frac{7aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{4(a^2-b^2)^2\,d\sqrt{e\sin(c+dx)}} - \frac{b\sqrt{e\sin(c+\cdots}}{2(a^2-b^2)\,de(a+b\cdots}$$

$$= -\frac{7aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{4(a^2-b^2)^2\,d\sqrt{e\sin(c+dx)}} + \frac{3a(5a^2+2b^2)\,\Pi\left(\frac{-\cdots}{b-\cdots}\right.}{8(-a^2+b^2\cdots}$$

$$= \frac{3\sqrt{b}\,(5a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8(-a^2+b^2)^{11/4}\,d\sqrt{e}} + \frac{3\sqrt{b}\,(5a^2+2\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 29.75, size = 1226, normalized size = 2.29

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] ((-1/2*b/((a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (7*a*b)/(4*(a^2 - b^2)^2*(a
 + b*Cos[c + d*x])))*Sin[c + d*x])/(d*Sqrt[e*Sin[c + d*x]]) + (Sqrt[Sin[c +
 d*x]]*((-14*a*b*Cos[c + d*x]^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*Ar
cTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan
[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2
 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c +
d*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c
```

```
+ d*x]] + b*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a
^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/
(-a^2 + b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)
*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b
^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*
x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2
, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[
c + d*x]^2)))))/((a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(8*a^2 + 6
*b^2)*Cos[c + d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*
(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*
ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[S
qrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I
*b*Sin[c + d*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 -
 b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
 + b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*Appel
lF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] -
2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-
a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*
Sin[c + d*x]^2)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x
]^2)))))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(8*(a - b)^2*(a
+ b)^2*d*Sqrt[e*Sin[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2837 vs. $2(559) = 1118$.

time = 0.59, size = 2838, normalized size = 5.30

method	result	size
default	Expression too large to display	2838

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-5/4*b^3*e/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2/(a^4-2*a^2*b^2+b^4)*(e*sin(d*
x+c))^(5/2)*a^2-1/2*b^5*e/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2/(a^4-2*a^2*b^2+
b^4)*(e*sin(d*x+c))^(5/2)-9/4*b*e^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2/(a^2-
b^2)*(e*sin(d*x+c))^(1/2)*a^2-1/2*b^3*e^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2
/(a^2-b^2)*(e*sin(d*x+c))^(1/2)-15/16*b*e/(a^4-2*a^2*b^2+b^4)*(e^2*(a^2-b^2
)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(
1/4)*(e*sin(d*x+c))^(1/2)+1)*a^2-3/8*b^3*e/(a^4-2*a^2*b^2+b^4)*(e^2*(a^2-b^
2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^
(1/4)*(e*sin(d*x+c))^(1/2)+1)-15/16*b*e/(a^4-2*a^2*b^2+b^4)*(e^2*(a^2-b^2)/
b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)-1)*a^2-3/8*b^3*e/(a^4-2*a^2*b^2+b^4)*(e^2*(a^2-b^2)
/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1
```

```
/4)*(e*sin(d*x+c))^(1/2)-1)-15/32*b*e/(a^4-2*a^2*b^2+b^4)*(e^2*(a^2-b^2)/b^
2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e
^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1
/2)))*a^2-3/16*b^3*e/(a^4-2*a^2*b^2+b^4)*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2
-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))
^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)
^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))-(cos(d*x+c)
^2*e*sin(d*x+c))^(1/2)*a*(3/2*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)+3/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Ell
ipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-15/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/
2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+3/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/
2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c
)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(
1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+15/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2
)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)
^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1
/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1
/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^
(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-4*a^2*(1/4*b^2/e/a^2/(a^2-b^2)*
(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)^2+1/16*b^2*(13*a^
2-6*b^2)/a^4/(a^2-b^2)^2/e*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2
*b^2+a^2)+13/32/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2
)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)
+1)^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-45/64/(a^2-b^2)^2/b/(-a^2+b^2)^(1/2)
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/
2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+9/16/a^2/(a^2-b^2)^2*b/(-a^2+b^2)^
(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*
x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1
)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^3/(-a^
2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/
(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d
*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+45/64/(a^2-b^2)^2/b/(-
a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin
(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-9/16/a^2/(a^2-b^2)^2
*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi(
```

```
(-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+3/16/a^4/(a^2-b
^2)^2*b^3/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*Elli
pticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))))/cos(d*
x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(e^(-1/2)/((b*cos(d*x + c) + a)^3*sqrt(sin(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e \sin (c+d x)} (a+b \cos (c+d x))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3),x)
[Out] int(1/((e*sin(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3), x)
```

3.86 $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=611

$$-\frac{5b^{3/2}(7a^2+2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}}+\frac{5b^{3/2}(7a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}}-\frac{}{2\,(a^2}$$

```
[Out] -5/8*b^(3/2)*(7*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(
1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d/e^(3/2)+5/8*b^(3/2)*(7*a^2+2*b^2)*arctanh
(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(13/4)/d
/e^(3/2)-1/2*b/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(1/2)-9/4*a*
b/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(1/2)+1/4*(5*b*(7*a^2+2*b
^2)-a*(8*a^2+37*b^2)*cos(d*x+c))/(a^2-b^2)^3/d/e/(e*sin(d*x+c))^(1/2)+5/8*a
*b*(7*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*
sin(d*x+c)^(1/2)/(a^2-b^2)^3/d/e/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+
5/8*a*b*(7*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+
1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1
/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3/d/e/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(
1/2)+1/4*a*(8*a^2+37*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1
/2)/(a^2-b^2)^3/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A]

time = 1.07, antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2773, 2943, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{5b^{3/2}(7a^2+2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8de^{3/2}\left(b^2-a^2\right)^{13/4}}-\frac{a(8a^2+37b^2)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{4de^2\left(a^2-b^2\right)^3\sqrt{\sin(c+dx)}}-\frac{9ab}{4de\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}\,(a+b\cos(c+dx))}-\frac{b}{2de\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}\,(a+b\cos(c+dx))^2}+\frac{5b(7a^2+2b^2)-a(8a^2+37b^2)\cos(c+dx)}{4de\left(a^2-b^2\right)^3\sqrt{e\sin(c+dx)}}-\frac{5ab(7a^2+2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8de\left(a^2-b^2\right)^3\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}-\frac{5ab(7a^2+2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8de\left(a^2-b^2\right)^3\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}+\frac{5b^{3/2}(7a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8de^{3/2}\left(b^2-a^2\right)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-5*b^(3/2)*(7*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(13/4)*d*e^(3/2)) + (5*b^(3/2)*(7*a^2
 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e
])])/(8*(-a^2 + b^2)^(13/4)*d*e^(3/2)) - b/(2*(a^2 - b^2)*d*e*(a + b*Cos[c
+ d*x])^2*Sqrt[e*Sin[c + d*x]]) - (9*a*b)/(4*(a^2 - b^2)^2*d*e*(a + b*Cos[c
 + d*x])*Sqrt[e*Sin[c + d*x]]) + (5*b*(7*a^2 + 2*b^2) - a*(8*a^2 + 37*b^2)*
Cos[c + d*x])/(4*(a^2 - b^2)^3*d*e*Sqrt[e*Sin[c + d*x]]) - (5*a*b*(7*a^2 +
2*b^2)*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/(8*(a^2 - b^2)^3*(b - Sqrt[-a^2 + b^2])*d*e*Sqrt[e*Sin[c +
d*x]]) - (5*a*b*(7*a^2 + 2*b^2)*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c
```

```
 - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*(a^2 - b^2)^3*(b + Sqrt[-a^2 +
b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (a*(8*a^2 + 37*b^2)*EllipticE[(c - Pi/2 +
 d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(4*(a^2 - b^2)^3*d*e^2*Sqrt[Sin[c + d*x]]
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
```

```
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
```

```
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{3/2}}\,dx = -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} - \frac{\int \frac{}{(a+b\,\text{co}}}{}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} - \frac{}{4\left(a^2-\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} - \frac{}{4\left(a^2-\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} - \frac{}{4\left(a^2-\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} - \frac{}{4\left(a^2-\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} - \frac{}{4\left(a^2-\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2\sqrt{e\sin(c+dx)}} - \frac{}{4\left(a^2-\right.}$$

$$= -\frac{5b^{3/2}(7a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{13/4}de^{3/2}} + \frac{5b^{3/2}(7a^2\ -}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 16.54, size = 922, normalized size = 1.51

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sin[c + d*x]^2*((-2*(-3*a^2*b - b^3 + a^3*Cos[c + d*x] + 3*a*b^2*Cos[c + d
*x])*Csc[c + d*x])/(a^2 - b^2)^3 + (b^3*Sin[c + d*x])/(2*(a^2 - b^2)^2*(a +
 b*Cos[c + d*x])^2) + (13*a*b^3*Sin[c + d*x])/(4*(a^2 - b^2)^3*(a + b*Cos[c
 + d*x]))))/(d*(e*Sin[c + d*x])^(3/2)) - (Sin[c + d*x]^(3/2)*(((8*a^3*b + 3
7*a*b^3)*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[
2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*S
qrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqr
t[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin
[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Si
n[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d*x
]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2))
+ (2*(8*a^4 + 72*a^2*b^2 + 10*b^4)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 -
 ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((
1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^
2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*
x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c
+ d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)) + (a*AppellF1[3/
4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c +
d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2]))/((a + b*Cos[
c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(8*(a - b)^3*(a + b)^3*d*(e*Sin[c + d
*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3430 vs. $2(631) = 1262$.
time = 0.71, size = 3431, normalized size = 5.62

method	result	size
default	Expression too large to display	3431

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (11/4/e*b^5/(a-b)^3/(a+b)^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c)
)^(7/2)*a^2+1/2/e*b^7/(a-b)^3/(a+b)^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*
sin(d*x+c))^(7/2)+15/4*e*b^3/(a-b)^3/(a+b)^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2
)^2*(e*sin(d*x+c))^(3/2)*a^4-13/4*e*b^5/(a-b)^3/(a+b)^3/(-b^2*cos(d*x+c)^2*
```

```
e^2+a^2*e^2)^2*(e*sin(d*x+c))^(3/2)*a^2-1/2*e*b^7/(a-b)^3/(a+b)^3/(-b^2*cos
(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(3/2)+35/32/e*b/(a-b)^3/(a+b)^3/(e^
2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*a^2*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e
^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1
/2)))+35/16/e*b/(a-b)^3/(a+b)^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*a^2*arcta
n(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+35/16/e*b/(a-b)
^3/(a+b)^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(e^2*(a^2-b
^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+5/16/e*b^3/(a-b)^3/(a+b)^3/(e^2*(a^2
-b^2)/b^2)^(1/4)*2^(1/2)*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(
d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)+(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+5/8/
e*b^3/(a-b)^3/(a+b)^3/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2
*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+5/8/e*b^3/(a-b)^3/(a+b)^3/(e^
2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*
sin(d*x+c))^(1/2)-1)+6/e*b/(a^2-b^2)^3/(e*sin(d*x+c))^(1/2)*a^2+2/e*b^3/(a^
2-b^2)^3/(e*sin(d*x+c))^(1/2)-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e*a*(b^2*(a
^2+3*b^2)/(a-b)^3/(a+b)^3*(-1/2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2
)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))
-1/2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+b^2*(a^2+3*b^2)/(a-b)^2/(
a+b)^2*(1/2*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2
)/(-cos(d*x+c)^2*b^2+a^2)-1/2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*E
llipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+
b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+
b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2
*2^(1/2))-3/8/(a^2-b^2)/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*si
n(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*Ell
ipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2
/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x
+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+4*a^2*b^2/(a-b)/(a+b)*(
1/4*b^2/e/a^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(
d*x+c)^2*b^2+a^2)^2+1/16*b^2*(11*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*sin(d*x+c)*(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-11/16/a^2/(a^2-b^2)
^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c
)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/8/a^
```

```
4/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1
/2))*b^2+11/32/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-21/64/(a^2-b^2)^2/b^2*(-sin(d*x+c)+1)
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^
(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^
2)^(1/2)/b),1/2*2^(1/2))+7/16/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2
+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/
2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2
)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))
-21/64/(a^2-b^2)^2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*Elliptic
Pi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(3/2),x)
```

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-3/2)/((b*cos(d*x + c) + a)^3*sin(d*x + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\sin\left(c+d\,x\right)\right)^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int(1/((e*sin(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3), x)
```

3.87 $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=629

$$\frac{7b^{5/2}(9a^2+2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}}+\frac{7b^{5/2}(9a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}}-\frac{\quad}{2\,(a^2-}$$

```
[Out] 7/8*b^(5/2)*(9*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1
/4)/e^(1/2))/(-a^2+b^2)^(15/4)/d/e^(5/2)+7/8*b^(5/2)*(9*a^2+2*b^2)*arctanh(
b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(15/4)/d/
e^(5/2)-1/2*b/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(3/2)-11/4*a*
b/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(3/2)+1/12*(7*b*(9*a^2+2*
b^2)-a*(8*a^2+69*b^2)*cos(d*x+c))/(a^2-b^2)^3/d/e/(e*sin(d*x+c))^(3/2)-1/12
*a*(8*a^2+69*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2
)^3/d/e^2/(e*sin(d*x+c))^(1/2)+7/8*a*b^2*(9*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/
2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d
*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^3/d/e^2/(a
^2-b*(b-(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)+7/8*a*b^2*(9*a^2+2*b^2)*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1
/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^
2-b^2)^3/d/e^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(e*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 1.15, antiderivative size = 629, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2773, 2943, 2945, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{7b^{5/2}(9a^2+2b^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8de^{5/2}\left(b^2-a^2\right)^{15/4}}+\frac{a(8a^2+69b^2)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{12de^2\left(a^2-b^2\right)^3\sqrt{e\sin(c+dx)}}-\frac{7ab^2(9a^2+2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8de^2\left(a^2-b^2\right)^3\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{e\sin(c+dx)}}-\frac{7ab^2(9a^2+2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8de^2\left(a^2-b^2\right)^3\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{e\sin(c+dx)}}-\frac{11ab}{4de\left(a^2-b^2\right)^2(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}(a+b\cos(c+dx))^2}+\frac{7b(9a^2+2b^2)-a(8a^2+69b^2)\cos(c+dx)}{12de\left(a^2-b^2\right)^3(e\sin(c+dx))^{3/2}}+\frac{7b^{5/2}(9a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8de^{5/2}\left(b^2-a^2\right)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (7*b^(5/2)*(9*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b
^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(15/4)*d*e^(5/2)) + (7*b^(5/2)*(9*a^2
+ 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e]
)])/(8*(-a^2 + b^2)^(15/4)*d*e^(5/2)) - b/(2*(a^2 - b^2)*d*e*(a + b*Cos[c +
 d*x])^2*(e*Sin[c + d*x])^(3/2)) - (11*a*b)/(4*(a^2 - b^2)^2*d*e*(a + b*Cos
[c + d*x])*(e*Sin[c + d*x])^(3/2)) + (7*b*(9*a^2 + 2*b^2) - a*(8*a^2 + 69*b
^2)*Cos[c + d*x])/(12*(a^2 - b^2)^3*d*e*(e*Sin[c + d*x])^(3/2)) + (a*(8*a^2
 + 69*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(12*(a^2 -
b^2)^3*d*e^2*Sqrt[e*Sin[c + d*x]]) - (7*a*b^2*(9*a^2 + 2*b^2)*EllipticPi[(2
*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*(
```

```
a^2 - b^2)^3*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*d*e^2*Sqrt[e*Sin[c + d*x]]) -
 (7*a*b^2*(9*a^2 + 2*b^2)*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*(a^2 - b^2)^3*(a^2 - b*(b + Sqrt[-a^2
 + b^2]))*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
```

```
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
```

```
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{5/2}}\,dx = -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} - \frac{\int \frac{}{(a+b\,c}}{}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} - \frac{}{4\left(a^2-\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} - \frac{}{4\left(a^2-\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} - \frac{}{4\left(a^2-\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} - \frac{}{4\left(a^2-\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} - \frac{}{4\left(a^2-\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{3/2}} - \frac{}{4\left(a^2-\right.}$$

$$= \frac{7b^{5/2}(9a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{15/4}de^{5/2}} + \frac{7b^{5/2}(9a^2+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 34.99, size = 1308, normalized size = 2.08

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] ((b^3/(2*(a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (15*a*b^3)/(4*(a^2 - b^2)^
3*(a + b*Cos[c + d*x])) - (2*(-3*a^2*b - b^3 + a^3*Cos[c + d*x] + 3*a*b^2*C
os[c + d*x])*Csc[c + d*x]^2)/(3*(a^2 - b^2)^3))*Sin[c + d*x]^3)/(d*(e*Sin[c
 + d*x])^(5/2)) + (Sin[c + d*x]^(5/2)*((2*(8*a^3*b + 69*a*b^3)*Cos[c + d*x]
^2*(a + b*Sqrt[1 - Sin[c + d*x]^2])*((a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqr
t[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Si
n[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^
2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] +
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]]))/(
4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*AppellF1[1/4, -1/2,
 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[Sin[c + d*
x]]*Sqrt[1 - Sin[c + d*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4,
Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4,
 -1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] + (a^2 -
b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a + b*Cos[c
 + d*x])*(1 - Sin[c + d*x]^2)) + (2*(8*a^4 - 120*a^2*b^2 - 42*b^4)*Cos[c +
d*x]*(a + b*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[b]*(2*ArcTan[1 -
((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1
 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2
] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x
]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c +
 d*x]] + I*b*Sin[c + d*x]]))/(-a^2 + b^2)^(3/4) + (5*a*(a^2 - b^2)*AppellF1
[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)]*Sqrt[
Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2,
1, 5/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*Appel
lF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2 + b^2)] +
(-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2
)/(-a^2 + b^2)])*Sin[c + d*x]^2)*(a^2 + b^2*(-1 + Sin[c + d*x]^2)))))/((a +
 b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(24*(a - b)^3*(a + b)^3*d*(e*S
in[c + d*x])^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3373 vs. $2(649) = 1298$.
time = 0.81, size = 3374, normalized size = 5.36

method	result	size

default	Expression too large to display	3374

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (13/4/e*b^5/(a-b)^3/(a+b)^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c)
)^(5/2)*a^2+1/2/e*b^7/(a-b)^3/(a+b)^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*
sin(d*x+c))^(5/2)+17/4*e*b^3/(a-b)^3/(a+b)^3/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2
)^2*(e*sin(d*x+c))^(1/2)*a^4-15/4*e*b^5/(a-b)^3/(a+b)^3/(-b^2*cos(d*x+c)^2*
e^2+a^2*e^2)^2*(e*sin(d*x+c))^(1/2)*a^2-1/2*e*b^7/(a-b)^3/(a+b)^3/(-b^2*cos
(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(1/2)+63/16/e*b^3/(a-b)^3/(a+b)^3*(
e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2
-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)*a^2+7/8/e*b^5/(a-b)^3/(a+b)^3*(e^2
*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^
2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)+63/16/e*b^3/(a-b)^3/(a+b)^3*(e^2*(a^2
-b^2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^
2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)*a^2+7/8/e*b^5/(a-b)^3/(a+b)^3*(e^2*(a^2-b^
2)/b^2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^
(1/4)*(e*sin(d*x+c))^(1/2)-1)+63/32/e*b^3/(a-b)^3/(a+b)^3*(e^2*(a^2-b^2)/b^
2)^(1/4)/(a^2*e^2-b^2*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e
^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1
/2)))*a^2+7/16/e*b^5/(a-b)^3/(a+b)^3*(e^2*(a^2-b^2)/b^2)^(1/4)/(a^2*e^2-b^2
*e^2)*2^(1/2)*ln((e*sin(d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/
2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(d*x+c)-(e^2*(a^2-b^2)/b^2)^(1/
4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2)))+2/e*b/(a^2-b^2)
^3/(e*sin(d*x+c))^(3/2)*a^2+2/3/e*b^3/(a^2-b^2)^3/(e*sin(d*x+c))^(3/2)-(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/e^2*a*(1/3*(-a^2-3*b^2)/(a^2-b^2)^3/(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+
2*cos(d*x+c)^2*sin(d*x+c))+b^2*(a^2+3*b^2)/(a-b)^3/(a+b)^3*(-1/2/b/(-a^2+b^
2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/2/b/(-a^2+b^2)^(1/2)*(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1
/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))+b^2*(a^2+3*b^2)/(a-b)^2/(a+b)^2*(1/2*
b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^
2)+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1
/2*2^(1/2))-5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+
b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2
```

```
*2^(1/2))+1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^
2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1
/2*2^(1/2))+5/8/(a^2-b^2)/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+
b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2
*2^(1/2))-1/4/a^2/(a^2-b^2)*b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^
2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1
/2*2^(1/2)))+4*a^2*b^2/(a+b)/(a-b)*(1/4*b^2/e/a^2/(a^2-b^2)*(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)^2+1/16*b^2*(13*a^2-6*b^2)/a^4/(a
^2-b^2)^2/e*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)+13/32
/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1
/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2
^(1/2))-3/16/a^4/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)
^(1/2),1/2*2^(1/2))*b^2-45/64/(a^2-b^2)^2/b/(-a^2+b^2)^(1/2)*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))
^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b
^2)^(1/2)/b),1/2*2^(1/2))+9/16/a^2/(a^2-b^2)^2*b/(-a^2+b^2)^(1/2)*(-sin(d*x
+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*
x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-
a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/16/a^4/(a^2-b^2)^2*b^3/(-a^2+b^2)^(1/2)*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),
1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+45/64/(a^2-b^2)^2/b/(-a^2+b^2)^(1/2)*
(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2
*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2
),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-9/16/a^...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-5/2)/((b*cos(d*x + c) + a)^3*sin(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int(1/((e*sin(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3), x)
```

3.88 $\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=700

$$-\frac{9b^{7/2}(11a^2+2b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{17/4}de^{7/2}}+\frac{9b^{7/2}(11a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{17/4}de^{7/2}}-\frac{}{2(}$$

[Out] -9/8*b^(7/2)*(11*a^2+2*b^2)*arctan(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(17/4)/d/e^(7/2)+9/8*b^(7/2)*(11*a^2+2*b^2)*arctanh(b^(1/2)*(e*sin(d*x+c))^(1/2)/(-a^2+b^2)^(1/4)/e^(1/2))/(-a^2+b^2)^(17/4)/d/e^(7/2)-1/2*b/(a^2-b^2)/d/e/(a+b*cos(d*x+c))^2/(e*sin(d*x+c))^(5/2)-13/4*a*b/(a^2-b^2)^2/d/e/(a+b*cos(d*x+c))/(e*sin(d*x+c))^(5/2)+1/20*(9*b*(11*a^2+2*b^2)-a*(8*a^2+109*b^2)*cos(d*x+c))/(a^2-b^2)^3/d/e/(e*sin(d*x+c))^(5/2)-3/20*(15*b^3*(11*a^2+2*b^2)+a*(8*a^4-64*a^2*b^2-139*b^4)*cos(d*x+c))/(a^2-b^2)^4/d/e^3/(e*sin(d*x+c))^(1/2)-9/8*a*b^3*(11*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^4/d/e^3/(b-(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-9/8*a*b^3*(11*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^4/d/e^3/(b+(-a^2+b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+3/20*a*(8*a^4-64*a^2*b^2-139*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^4/d/e^4/sin(d*x+c)^(1/2)

Rubi [A]
time = 1.35, antiderivative size = 700, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2773, 2943, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{9b^{7/2}(11a^2+2b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8de^{7/2}\left(b^2-a^2\right)^{17/4}}-\frac{13ab}{4de\left(a^2-b^2\right)^2(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))}-\frac{b}{2de\left(a^2-b^2\right)(e\sin(c+dx))^{5/2}(a+b\cos(c+dx))^2}+\frac{9b(11a^2+2b^2)-a(8a^2+109b^2)\cos(c+dx)}{20de\left(a^2-b^2\right)^3(e\sin(c+dx))^{5/2}}+\frac{9b^{7/2}(11a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{b^2-a^2}}\right)}{8de^{7/2}\left(b^2-a^2\right)^{17/4}}+\frac{9ab^3(11a^2+2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8de^3\left(a^2-b^2\right)^4\left(b-\sqrt{b^2-a^2}\right)\sqrt{e\sin(c+dx)}}+\frac{9ab^3(11a^2+2b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{8de^3\left(a^2-b^2\right)^4\left(\sqrt{b^2-a^2}+b\right)\sqrt{e\sin(c+dx)}}-\frac{3a(8a^4-64a^2b^2-139b^4)\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{20de^4\left(a^2-b^2\right)^4\sqrt{\sin(c+dx)}}-\frac{3(15b^3(11a^2+2b^2)+a(8a^4-64a^2b^2-139b^4)\cos(c+dx))}{20de^3\left(a^2-b^2\right)^4\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2)),x]

[Out] (-9*b^(7/2)*(11*a^2 + 2*b^2)*ArcTan[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(17/4)*d*e^(7/2)) + (9*b^(7/2)*(11*a^2 + 2*b^2)*ArcTanh[(Sqrt[b]*Sqrt[e*Sin[c + d*x]])/((-a^2 + b^2)^(1/4)*Sqrt[e])])/(8*(-a^2 + b^2)^(17/4)*d*e^(7/2)) - b/(2*(a^2 - b^2)*d*e*(a + b*Cos[c + d*x])^2*(e*Sin[c + d*x])^(5/2)) - (13*a*b)/(4*(a^2 - b^2)^2*d*e*(a + b*Cos[c + d*x])*(e*Sin[c + d*x])^(5/2)) + (9*b*(11*a^2 + 2*b^2) - a*(8*a^2 + 109*b^2)*Cos[c + d*x])/(20*(a^2 - b^2)^3*d*e*(e*Sin[c + d*x])^(5/2)) - (3*(15*b^3*(11*a^2 + 2*b^2) + a*(8*a^4 - 64*a^2*b^2 - 139*b^4)*Cos[c + d*x]))/(

```
20*(a^2 - b^2)^4*d*e^3*Sqrt[e*Sin[c + d*x]]) + (9*a*b^3*(11*a^2 + 2*b^2)*El
lipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
d*x]])/(8*(a^2 - b^2)^4*(b - Sqrt[-a^2 + b^2])*d*e^3*Sqrt[e*Sin[c + d*x]])
+ (9*a*b^3*(11*a^2 + 2*b^2)*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (c - P
i/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(8*(a^2 - b^2)^4*(b + Sqrt[-a^2 + b^2]
)*d*e^3*Sqrt[e*Sin[c + d*x]]) - (3*a*(8*a^4 - 64*a^2*b^2 - 139*b^4)*Ellipti
cE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(20*(a^2 - b^2)^4*d*e^4*Sqr
t[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
```

```
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^3(e\sin(c+dx))^{7/2}}\,dx = -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{\int}{(a+ \cdots}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{\cdots}{4\left(a^2 \cdots\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{\cdots}{4\left(a^2 \cdots\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{\cdots}{4\left(a^2 \cdots\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{\cdots}{4\left(a^2 \cdots\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{\cdots}{4\left(a^2 \cdots\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{\cdots}{4\left(a^2 \cdots\right.}$$

$$= -\frac{b}{2\left(a^2-b^2\right)de(a+b\cos(c+dx))^2(e\sin(c+dx))^{5/2}} - \frac{\cdots}{4\left(a^2 \cdots\right.}$$

$$= -\frac{9b^{7/2}(11a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{e\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}\sqrt{e}}\right)}{8\left(-a^2+b^2\right)^{17/4}de^{7/2}} + \frac{9b^{7/2}(11 \cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 16.65, size = 1014, normalized size = 1.45

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] (Sin[c + d*x]^4*((-2*(50*a^2*b^3 + 10*b^5 + 3*a^5*Cos[c + d*x] - 24*a^3*b^2
*Cos[c + d*x] - 39*a*b^4*Cos[c + d*x])*Csc[c + d*x])/(5*(a^2 - b^2)^4) - (2
```

```
*(-3*a^2*b - b^3 + a^3*Cos[c + d*x] + 3*a*b^2*Cos[c + d*x])*Csc[c + d*x]^3)
/(5*(a^2 - b^2)^3) - (b^5*Sin[c + d*x])/(2*(a^2 - b^2)^3*(a + b*Cos[c + d*x
])^2) - (21*a*b^5*Sin[c + d*x])/(4*(a^2 - b^2)^4*(a + b*Cos[c + d*x]))))/(d
*(e*Sin[c + d*x])^(7/2)) - (3*Sin[c + d*x]^(7/2)*(((8*a^5*b - 64*a^3*b^3 -
139*a*b^5)*Cos[c + d*x]^2*(3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqr
t[2]*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]
*Sqrt[b]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*Sin[c + d*x]] + Log[S
qrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + b*S
in[c + d*x]]) + 8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*
Sin[c + d*x]^2)/(-a^2 + b^2)]*Sin[c + d*x]^(3/2))*(a + b*Sqrt[1 - Sin[c + d
*x]^2]))/(12*b^(3/2)*(-a^2 + b^2)*(a + b*Cos[c + d*x])*(1 - Sin[c + d*x]^2)
) + (2*(8*a^6 - 64*a^4*b^2 - 304*a^2*b^4 - 30*b^6)*Cos[c + d*x]*(((1/8 + I/
8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] -
 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Lo
g[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*b*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(
1/4)*Sqrt[Sin[c + d*x]] + I*b*Sin[c + d*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))
+ (a*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (b^2*Sin[c + d*x]^2)/(-a^2
+ b^2)]*Sin[c + d*x]^(3/2))/(3*(a^2 - b^2)))*(a + b*Sqrt[1 - Sin[c + d*x]^2
]))/((a + b*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(40*(a - b)^4*(a + b)
^4*d*(e*Sin[c + d*x])^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3708 vs. $2(716) = 1432$.

time = 0.87, size = 3709, normalized size = 5.30

method	result	size
default	Expression too large to display	3709

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-19/4/e^3*b^7/(a-b)^4/(a+b)^4/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x
+c))^(7/2)*a^2-1/2/e^3*b^9/(a-b)^4/(a+b)^4/(-b^2*cos(d*x+c)^2*e^2+a^2*e^2)^
2*(e*sin(d*x+c))^(7/2)-23/4/e*b^5/(a-b)^4/(a+b)^4/(-b^2*cos(d*x+c)^2*e^2+a^
2*e^2)^2*(e*sin(d*x+c))^(3/2)*a^4+21/4/e*b^7/(a-b)^4/(a+b)^4/(-b^2*cos(d*x+
c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(3/2)*a^2+1/2/e*b^9/(a-b)^4/(a+b)^4/(-b^
2*cos(d*x+c)^2*e^2+a^2*e^2)^2*(e*sin(d*x+c))^(3/2)-99/32/e^3*b^3/(a-b)^4/(a
+b)^4/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*a^2*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(
d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2)))-99/16/e^3*b^3/(a-b)^4/(a+b)^4/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1
/2)*a^2*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)-99
/16/e^3*b^3/(a-b)^4/(a+b)^4/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*a^2*arctan(2^
```

```
(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)-9/16/e^3*b^5/(a-b)^
4/(a+b)^4/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*ln((e*sin(d*x+c)-(e^2*(a^2-b^2)
/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2)/b^2)^(1/2))/(e*sin(
d*x+c)+(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)*2^(1/2)+(e^2*(a^2-b^2
)/b^2)^(1/2)))-9/8/e^3*b^5/(a-b)^4/(a+b)^4/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(e^2*(a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)+1)-9/8/e^3*
b^5/(a-b)^4/(a+b)^4/(e^2*(a^2-b^2)/b^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(e^2*(
a^2-b^2)/b^2)^(1/4)*(e*sin(d*x+c))^(1/2)-1)+6/5/e*b/(a+b)^3/(a-b)^3/(e*sin(
d*x+c))^(5/2)*a^2+2/5/e*b^3/(a+b)^3/(a-b)^3/(e*sin(d*x+c))^(5/2)-20/e^3*b^3
/(a-b)^4/(a+b)^4/(e*sin(d*x+c))^(1/2)*a^2-4/e^3*b^5/(a-b)^4/(a+b)^4/(e*sin(
d*x+c))^(1/2)-(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/e^3*a*(-6*b^4*(a^2+b^2)/(a-
b)^4/(a+b)^4*(-1/2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*Elliptic
Pi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-1/2/b^2*(-si
n(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*s
in(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/
(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2)))-1/5*(-a^2-3*b^2)/(a^2-b^2)^3/(cos(d*x+
c)^2*e*sin(d*x+c))^(1/2)/sin(d*x+c)/(cos(d*x+c)^2-1)*(6*(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2
)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+6*cos(d*x+c)^4*sin(d*x+c)-8*
cos(d*x+c)^2*sin(d*x+c))-b^4*(5*a^2+3*b^2)/(a-b)^3/(a+b)^3*(1/2*b^2/e/a^2/(
a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^
2)-1/2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1
/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1
)^(1/2),1/2*2^(1/2))-3/8/(a^2-b^2)/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(
1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/
2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)^(1/2)/b)*EllipticP
i((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))-3/8/(a^2-b^2)
/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)
^(1/2),1/(1+(-a^2+b^2)^(1/2)/b),1/2*2^(1/2))+1/4/a^2/(a^2-b^2)*(-sin(d*x+c)
+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)/(1+(-a^2+b^2)^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(-a^2
+b^2)^(1/2)/b),1/2*2^(1/2)))-4*a^2*b^4/(a-b)^2/(a+b)^2*(1/4*b^2/e/a^2/(a^2-
b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)^2
+1/16*b^2*(11*a^2-6*b^2)/a^4/(a^2-b^2)^2/e*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)/(-cos(d*x+c)^2*b^2+a^2)-11/16/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/8/a^4/(a^2-b^2)^2*(-sin
(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*si
```

```
n(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2+11/32/a^2/
(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2
))-3/16/a^4/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2
),1/2*2^(1/2))*b^2-21/64/(a^2-b^2)^2/b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(-a^2+b^2)
^(1/2)/b)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(-a^2+b^2)^(1/2)/b),1/2*2^(
1/2))+7/16/a^2/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**3/(e*sin(d*x+c))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(e^(-7/2)/((b*cos(d*x + c) + a)^3*sin(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,\sin\left(c+d\,x\right)\right)^{7/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3),x)
[Out] int(1/((e*sin(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section 480
4.2 Listing of Grading functions 480

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```